



Eighteen Flavors

A Lesson for Seventh and Eighth Graders

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"Eighteen Flavors," a poem in Shel Silverstein's Where the Sidewalk Ends (HarperCollins, 1974), describes the flavors of an ice-cream cone piled high with eighteen scoops. It ends in disappointment when the scoops topple to the ground. In this lesson, students measure the height of a paper ice-cream cone with one scoop of ice cream, then two, three, four, five, and so on. They try to find a general approach for determining the height of a cone with any number of scoops. Then, using n to represent any number of scoops, students try writing an equation for the height of the cone. This lesson is from the new book Math and Literature, Grades 6–8, by Jennifer M. Bay-Williams and Sherri L. Martinie (Math Solutions Publications, 2004).

When I read "Eighteen Flavors" aloud, I asked the class why there was a "sniffle" at the end of the poem.

"Because you can't eat ice cream off the ground!" Lori said.

"Does it surprise you that these eighteen scoops would be on the ground?" I asked.

"No, because that's too many for the cone to hold up," Michael answered.

I held up a sheet with a cone drawn on it along with circles to represent scoops of ice cream (see Blackline Masters at end of lesson). "If the scoops were as big as the circles on this sheet, how tall do you think eighteen scoops of ice cream on a cone would be? Using a sheet of these cones and scoops, try to answer the following questions: How tall is an ice-cream cone with one scoop of ice cream? With two scoops? Three scoops? Four scoops? Five? Eighteen? Twenty? Fifty? N scoops?" I recorded the questions on the board:

How tall is an ice-cream cone with 1 scoop of ice cream?

2 scoops?

3?

4?

5?

18?

20?

50?

n ?

"What is n ?" I asked.

"It's a variable," John replied.

"Why am I writing a variable to refer to scoops of ice cream?" I questioned.

"You're using the variable because the number of scoops can vary. Each time you go to the

ice-cream store, it can be different,” John responded.

I organized the students into groups of four and distributed ice-cream shapes and scissors to each group. Having to cut out the cones and circles gave the students something to do as they began to talk about solving the problem. Students soon asked for rulers and began measuring the cones and the scoops.

All but one group decided to have the scoops overlap on the cone because, as Mark explained, “that’s how it would really be.”

“Do we want to build a table or do something else?” Lori asked.

“Do you just want to measure the scoops and look for a pattern to start with? If we can find the pattern to start with then maybe we won’t have to make the table,” Kathy suggested.

I circulated as the students worked and listened to their conversations. In one group, I overheard Allison say, “To figure out the height of each scoop when they overlap, let’s just say we would take half an inch off of each one.”

“So each scoop measures four inches and we take off one-half inch, making each scoop three and a half inches,” Amy said.

“The cone is seven inches,” James added.

“One scoop and one cone would be ten and a half inches,” Allison replied. Allison, Amy, and James began to write this information down in a table each had created on notebook paper; only Eric did not begin writing.

“Let me see yours,” James asked Amy as he tried to record the data in his table.

“Two scoops and one cone would be fourteen inches . . . seven plus three and a half plus three and a half,” Allison said as she thought out loud.

“And three scoops would be seventeen and a half. That’s three and a half multiplied by three and then you add on seven,” Amy said as everyone in the group except Eric recorded this diligently on his or her paper.

“Eighteen scoops would equal seventy inches. See if that makes sense,” Allison insisted as she handed her notebook to Eric, trying to get him more involved.

“Yeah, eighteen times three and a half is sixty-three, and add on the seven-inch cone to get seventy,” he responded as he checked her work.

“For twenty, just do the same thing,” Amy suggested.

“Or just add on seven more inches for those two more scoops,” commented James, who had seemed, until this time, somewhat unsure of what everyone was doing and had continued to ask to see what other members in his group were writing down on their papers. “Then to find fifty scoops on a cone, if twenty scoops is seventy-seven inches, then double that for twenty more scoops, and then find out how much ten scoops is,” he continued.

“That won’t work because the cone is seven inches of that total, and we would be adding in

more than one cone,” Allison insisted.

“We did something wrong,” Amy said.

“Let’s go back. Five scoops would be five times three and a half, which is seventeen and a half, plus seven is twenty-four and a half,” Allison calculated. “So fifty times three and a half plus seven should work to find the height of fifty scoops if that works for five.”

“I get it,” Eric said. “We use three and one-half each time and multiply by the number of scoops. I don’t get how to use the variable though.”

This group tried without much success to figure out how to write the equation using the variable. Although the students could find and describe the pattern, they were unable to express it using the variable. However, they were helped by the whole-class discussion that followed.

Lori, Sue, Mark, and Paul began by measuring the cone with one scoop on it and writing down the measurement. They then added another scoop and measured again. They overlapped scoops but measured in centimeters, not in inches.

“Now we can subtract to find out how much that next scoop was,” suggested Lori.

“It would be six centimeters,” Sue responded as she measured the cone and scoops.

“When we add them on, all the scoops will be the same, except the top one will have a little more because it is not as squished,” Lori noticed. “We can use the measurement of the cone and the first scoop together and then add on the extra scoops because each extra scoop would add on six centimeters. To find eighteen scoops, you have to take six centimeters times seventeen and then add on the twenty-five from the beginning part and it gives us one hundred twenty-seven centimeters.”

“So you add all the ice-cream scoops in between except for the top scoop and the cone and you add those on last,” Sue repeated. “But I don’t know how to explain it on paper.”

Mark spoke up. “The pattern is that you add six each time you have a scoop, but if you skip one then you can add on the cone with one scoop on it in the end so you can include that little bit extra. So it would be n minus one times six plus twenty-five,” Mark stated. He had written: $(n - 1) \times 6 + 25$.

“This is confusing,” Sue shared.

“Yes, but it’ll get worse as you move on using equations unless you try to figure it out now,” Lori said. She went over their reasoning again to help Sue really understand the equation.

Another group started by placing the scoops on their desks and deciding how much each scoop was to be overlapped by the next and deciding how to measure the cone and the scoops.

“If you know what one scoop is, how do you figure out how much eighteen scoops is?” Andy asked.

“Multiply,” Joy responded.

“How are we going to find the height of the scoops?” Brad asked.

"We can measure the diameter of a scoop," Andy responded.

"Yeah, and then after we multiply the diameter by the number of scoops, we could add on the height of the cone," added Joy.

Each of them went to work independently drawing a table, measuring, and recording his or her numbers in the table.

"Maybe it would be easier to take the overlap off first," Andy suggested.

"What do you mean?" Joy asked.

"Well," Andy said, "to find the height of three scoops on a cone, I was doing three times four inches, the height of the scoops, minus three times one-half, the size of the overlap for each scoop. It seems like it would be easier to take four minus one-half and get three and one-half and then multiply that by the number of scoops so you don't have to subtract it later."

"That's what I was doing already," Joy said. "Then you just have to add the height of the cone to the scoops."

"Oh," Andy said as he began erasing his notebook page.

They continued working quietly, occasionally checking numbers with each other. Finally, Brad asked, "I don't get what to do with n . What number is it?"

"It could be any number," explained Andy. "It's a variable and we use it to help explain the pattern that we are using to find the height of the different ice-cream cones. Ours would be three and a half times n plus seven because every time we did the number of scoops, that is n , times three and one-half and then added on the cone, seven."

In the end, almost everyone understood how to find the height of a cone with any number of scoops piled on top. Many struggled, however, with writing an equation with a variable. After allowing the groups time to discover the pattern and discuss possible equations, we shared as a class what we had found.

Paul began, "We multiplied the number of scoops by three and a half."

"What about for n scoops?" I asked the class.

"The same thing you did to all the others—multiply by three and a half," Allison responded.

"But there is no number here to multiply by three and a half, so what should I do?" I asked.

" N could be any number, so you just write down to multiply it by three and a half. Instead of having two times three and a half, and getting an answer, you would have n times three and a half and it would just stay that way," Steve replied.

"How were you figuring out your total then?" I inquired.

"By multiplying the number of scoops by three and a half and then adding on seven for the cone. So the equation will end up being the number of scoops times three and a half plus seven," Andy explained.

"The variable means I can change the number of scoops but still do the same thing to find out how tall the ice-cream cone and scoops are. In real life, ice-cream scoops will be different sizes. We could make an equation for today's scoops because we made them so that they are all the same size," I explained.

We went on to discuss other strategies that students used and other equations that they discovered. After this discussion, I gave the students time to write in their notebooks about their solution strategies and equations and to complete their tables or lists if they hadn't already. (See Figures 1 and 2.)

	1	2	3	5	18	20	50	11		cone - 7 in.
scoops	3	6	9	15	54	60	150	11		scoop - 4 3
cones	7	7	7	7	7	7	7	7		overlap
total	10	13	16	22	61	67	157	12		

They are 3 in because the scoop each other

The pattern is take the number by 3

and you get your scoops. The cones

were 7 in. When I got the scoops

I took 7 + the number of scoops

I got and that was my answer.

Ex. $150 + 7 = 157$

50 scoops + cone

I did this because the cones

were 7 in. + the scoops were 3

Figure 1. Alan's group considered the "squishing" of the scoops on the cone. The students subtracted a half inch from the top and bottom of each scoop to account for the overlap of the scoops when stacked on top of one another.

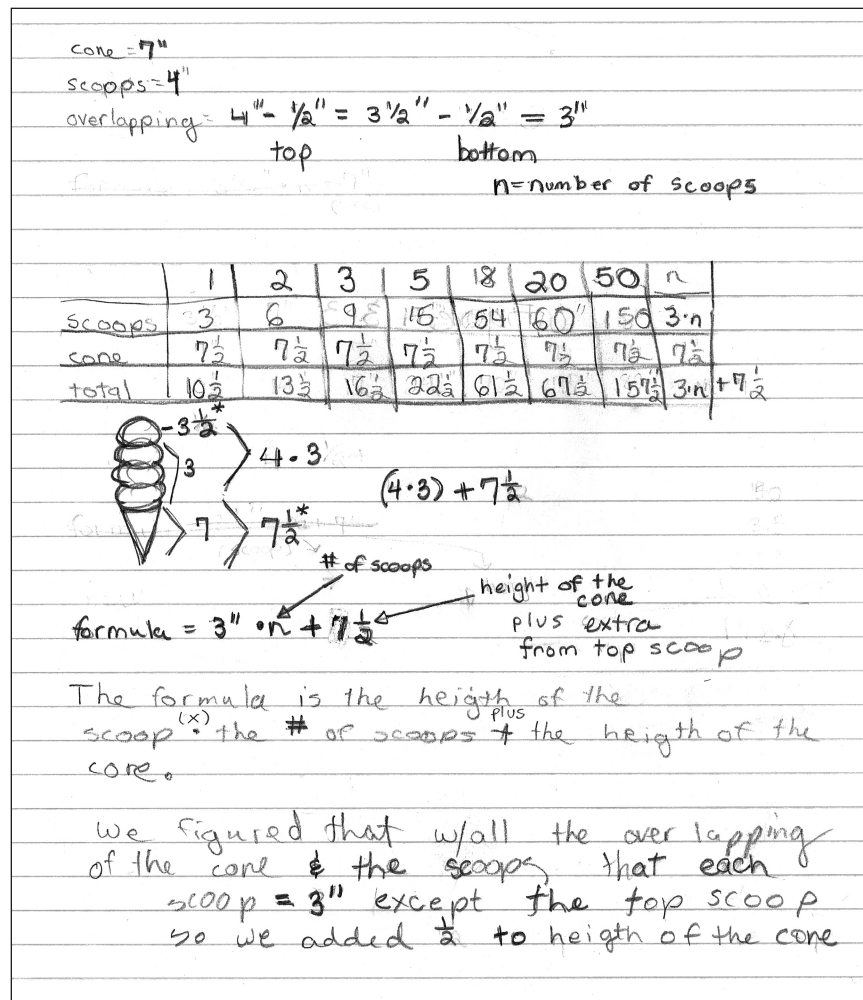


Figure 2. Upon realizing that the top scoop would be squished only on the bottom, Ron added an extra half inch to account for this in the height of the cone for the equation he came up with.

Eighteen Flavors Ice-Cream Shapes

