CHAPTER 1

Two of Everything

A First Experience with Growth Patterns

OVERVIEW

The children's book *Two of Everything*, by Lily Toy Hong, tells the story of a magical brass pot that doubles whatever is put into it. The story is an engaging context for providing students with a beginning experience with examining a growth pattern, recording and extending data on a T-chart, and representing the pattern algebraically with an equation. The experience is then extended by changing the doubling rule of the pot to other rules for the children to figure out. The students use T-charts to represent what goes into and comes out of the pot and describe the patterns with both words and equations. Students also create rules of their own for others to guess.

BACKGROUND

Two of Everything, by Lily Toy Hong, is a Chinese folktale about an elderly couple, Mr. and Mrs. Haktak, who find magic in an ancient brass pot. Mr. Haktak discovers the pot while digging in his garden, and he puts his coin purse into it for safekeeping. When he brings the pot into the house for Mrs. Haktak, she accidentally drops her hairpin into the pot. When she reaches in to get it, she pulls out two hairpins and two coin purses! Mr. and Mrs. Haktak realize their good luck and get to work doubling their possessions. The story takes a hilarious turn when Mrs. Haktak loses her balance and falls into the pot.

This lesson uses the model of the magical pot to introduce students to the idea of functions. A function is a relationship between two variables in which the value of one variable, often called the *output*, depends on the value of the other, often called the *input*. An important characteristic of a function is that for every input, there is exactly one output. When the students identify a number that is an input value, the rule for the function pairs that number with exactly one other number, the output value.

For example, if a function rule is to add two to the input number, and the input number is six, then eight is the only possible output number for that input. Using a triangle, $\triangle$, and a box, $\square$, as symbols to represent the variables, an equation to describe this function rule could be $\triangle = \square + 2$. In this lesson, the students use boxes and triangles as the variables to describe the patterns they investigate. In other lessons, they use other variables as well; for example, the equation △ = □ + 2 can also be written in others ways and using other variables: O = $I + 2$, $x + 2 = y$, $y = x + 2$, and so on. Over time, students become comfortable with different algebraic representations and learn how these representations connect to their related T-charts.

In later grades, students study functions in more depth. In grades 3–5, however, our goal is introductory and, therefore, we don't identify the activity to the class as an investigation of functions or present a formal definition of *function* to the students. Rather, we keep the focus of the lesson on having students create and interpret rules, represent them numerically on a T-chart, and describe them first with words and then algebraically with equations.

If you're not familiar with functions, check the Introduction for background information about the mathematics.

VOCABULARY

equation, input value, output value, T-chart, variable

MATERIALS

- *Two of Everything*, by Lily Toy Hong (Morton Grove, IL: Albert Whitman & Company, 1993)

TIME

- at least three class periods

The Lesson

Day 1

"Today I'd like to share a book with you written by Lily Toy Hong," I said. "It's called *Two of Everything*."

"We read that book last year," Michael said. "It was funny." Several others nodded their heads, indicating their agreement with Michael.

"How many of you have heard the story before?" I asked. Several students raised their hands. "Do you recommend it for other students?"

"Yes!" they chorused.

I read the story aloud. The class giggled when Mrs. Haktak fell into the pot and became two Mrs. Haktaks rather than one. Sam said, "I don't think my dad would want two wives!"

Audrey commented, "I wonder what Mr. Haktak will do."

I continued reading, and when Mr. Haktak also fell into the pot, Karly squealed, "Oh no! Now what?"

James said, "Each Mr. Haktak can have a Mrs. Haktak!"

When I finished reading the story, Andrea said, "It's neat the way the pot gave Mr. and Mrs. Haktak friends and everything everybody needed."

I then drew a T-chart on the board and labeled the columns *In* and *Out*. "Here's a way we can keep track of what goes into and comes out of the pot," I explained to the students. I wrote *5* about halfway down the In column of the T-chart, leaving room to record 1, 2, 3, and 4 above it.

"Suppose we put five coins in the pot. How many would come out?" I asked.

"Ten," Audrey replied.

"How do you know?" I asked. I always encourage the students to explain how they arrived at their answer.

"In the story, things that fell into the pot doubled," Audrey explained. "If five coins went in, then five would double, which is five plus five. That makes ten coins." The other students nodded their agreement with Audrey.

"Does anyone have something to add?" I asked. No one did. I wrote *10* on the T-chart in the right column next to the *5*.

In	Out
5	10

"What about four coins? If we put in four coins, how many would come out?" The students raised their hands immediately. "Use your fingers to show me how many coins would come out." The students quickly showed eight fingers.

"Can I share how I got it?" Annie asked. I nodded.

"I know you could just add four and four, but I did four times two and got eight," Annie shared. I added *4* and *8* to the T-chart above where I had recorded *5* and 10.

"What if we put in three coins?" I continued, writing *3* above the 4 in the In column. "What is the output value if the input value is three?" I used the terminology *output value* and *input value* in this instance to help the children become familiar with this alternative terminology for *in* and *out*, and I used the terminology interchangeably throughout the lesson.

"Six," the class responded. I added *6* to the chart.

I then asked the children a question that required them to think differently about the information we were collecting. "What if four coins came out of the pot? How many coins would we have to put in so that four coins would come out?"

Armando said, "If four came out, then two went in. Two plus two equals four. It works!"

"Or you could do times to check instead of addition," Brianna added. "Two times two is like two plus two, and it equals four." I added *2* and *4* to the T-chart.

In order to emphasize that doubling a number can be interpreted both as multiplying it by two and as adding it to itself, I said, "I agree with Brianna that we can use either multiplication or addition to think about the number of coins. Why is that?"

Rick said. “Sometimes multiplication can be used as a shorter way to add, like when all the numbers to be added are the same.”

“Would someone else like to share their thoughts?” I asked.

“Doubling is like adding the same number twice or multiplying a number by two, right?” James asked to clarify his own understanding. I nodded. No one had anything else to add.

I returned to the T-chart and said, “If the input is one coin, how many coins will come out?”

“Two,” the class responded. I added *1* and *2* to the chart.

In	Out
1	2
2	4
3	6
4	8
5	10

Discussing the Patterns in the T-Chart James, a student who loved to look for patterns, commented on the T-chart. “I see a pattern,” he said. “The In numbers are going up by one each time—one, two, three, four, five. And the Out numbers are counting by twos—two, four, six, eight, ten.” (If James hadn’t made his comment, I would have initiated a discussion at this point in the lesson by asking the students what patterns they noticed in the columns of numbers.)

“Oh yeah,” a few students added softly.

I wrote James’s idea on the board. Recording students’ ideas not only validates their thinking but also provides the others with a reference of their classmates’ ideas. In addition, taking the time to record James’s idea gave the other students the opportunity to consider what he said.

James
The In numbers go up by ones (1, 2, 3) and the Out numbers count by twos (2, 4, 6).

“Does everyone agree with James’s pattern?” I asked. The students indicated they did. “Who sees a different pattern?”

“If you multiply the In number by two, you get the Out number,” Brianna shared. “One times two is two, two times two is four, three times two is six.”

I recorded Brianna’s pattern on the board:

Brianna
$1 \times 2 = 2$
$2 \times 2 = 4$
$3 \times 2 = 6$

“If you agree with Brianna’s pattern please put your thumb up, if you disagree put your thumb down, and if you’re not sure put your thumb sideways,” I said. The students indicated their agreement by putting their thumbs up.

“Does anyone see another pattern?” I asked. Nina raised her hand.

"I noticed something," Nina began. "You add the In number to itself, and it gives the Out number. One plus one is two, two plus two is four, three plus three is six." I recorded Nina's idea on the board as she explained it:

Nina
1 + 1 = 2
2 + 2 = 4
3 + 3 = 6

"What would come next in Nina's pattern?" I asked.
"Four plus four is eight," Beatriz said.
"Next?" I asked after adding Beatriz's idea to Nina's pattern.
"Five plus five equals ten," David answered.
"I know the next one," Andrea said. "Six plus six equals twelve." I added David's and Andrea's contributions to Nina's pattern:

Nina
1 + 1 = 2
2 + 2 = 4
3 + 3 = 6
4 + 4 = 8
5 + 5 = 10
6 + 6 = 12

"And what if we put ten coins into the pot?" I asked.
"I know the answer," Beatriz said. "It's twenty because ten plus ten is twenty."
"Who agrees?" I asked the class. All hands went up.
"I thought of it differently, though," David said. "I did ten times two, and that's twenty."
"That's like Brianna's pattern," Karly noticed.
"Anyone think of it differently?" I asked.
"I think it could be two times ten," Annie said.
"Let's see what we have so far," I said. As I wrote on the board, I explained, "Here's how Beatriz, David, and Annie thought of it."

10 + 10 = 20 Beatriz
10 × 2 = 20 David
2 × 10 = 20 Annie

"David's and Annie's ways are the same," James said.
"No, they aren't," Armando answered. "They equal the same amount of things, but they don't look the same."
"I don't get it," Kris said.
"Well, two groups of ten look different than ten groups of two," Armando explained.
"Oh, I get it now," Kris said.
"I agree with Armando," I said. "The answer is the same, but they do look different. Let's look back at the chart." As the children watched, I added *10* to the In column of the T-chart, leaving some space so that I could later fill in the numbers from 6 through 9. I also wrote *20* in the Out column.
"It's missing some numbers," Andrea noticed.
"It is," I agreed. "Let's fill in those numbers together." I wrote *6, 7, 8,* and *9* in the In column and then had children tell me what to write in the

Out column for each. When we had filled in the T-chart up to 10, I wrote three dots following the last number in each column and then wrote *100* in

In	Out
1	2
2	4
3	6
4	8
5	10
6	12
7	14
8	16
9	18
10	20
.	.
.	.
.	.
100	

the In column.

"The dots mean that I've purposely skipped some numbers," I explained. "What is the output value if the In column says we put one hundred coins into the pot?"

"Two hundred," Rick said.

"How did you figure that?" I asked.

"I multiplied one hundred times two and that's two hundred," Rick explained. I wrote on the board:

Rick
100 × 2 = 200

"I did it differently," Andrea said. "I did one hundred plus one hundred equals two hundred."

I wrote on the board:

Andrea
100 + 100 = 200

"You can do the multiplication the other way, so it's two times one hundred," Beatriz said. I wrote Beatriz's idea on the board:

Beatriz
2 × 100 = 200

No one had anything else to say. I added *200* to the T-chart and asked, "Suppose one hundred coins came out of the pot. How many coins went in?"

"Fifty!" Audrey said.
"Fifty!" Cami agreed.
"Does anyone have a different answer?" I asked. No one did.
"Cami, can you explain?" I asked.
"You have to take one-half of the Out number to find the In number," Cami explained.
"I already knew that fifty plus fifty is one hundred," Nina said.
"Hey, taking half of something is making it into two equal groups. If you add those two equal groups, it's like doubling," Sam said with surprise. "It's, like, all connected together!" Sam's delighted surprise reminded me that children need many opportunities to make connections among ideas.

Generalizing from the T-Chart I redirected the students' attention to Nina's pattern, which was still recorded on the board:

Nina
1 + 1 = 2
2 + 2 = 4
3 + 3 = 6
4 + 4 = 8
5 + 5 = 10
6 + 6 = 12

"Who can describe Nina's pattern for figuring out the output when you know the number of coins put into the pot?" I asked.
"Take one number and add two of the number," Armando said.
"What number?" I pushed.
"The In number," Armando added.
"And what do I add to the In number?" I pushed further.
"Itself," Armando said. I wrote Armando's idea on the board as follows:

The Out number is equal to the In number added to itself.

While Armando didn't offer a complete sentence, I used a complete sentence to record his idea. The students would soon translate the sentences to equations, so complete sentences were necessary.
"You could say it another way," Annie suggested. "You could say In plus In equals Out." I recorded Annie's suggestion.

In plus In equals Out.

"Does anyone have another idea?" I asked. No one did, so I said, "It looks like we have two different ways to describe Nina's pattern."
I then pointed to Brianna's pattern on the board and said, "What about Brianna's pattern? How could you describe Brianna's pattern?"

Brianna
1 × 2 = 2
2 × 2 = 4
3 × 2 = 6

"Brianna's way is to times the In number by two," Sam said.
"So to get the Out number, you do the In number times two?" I asked. Sam nodded. I wrote on the board:

The Out number is equal to the In number times two.

I then said to the class, "A shortcut way to describe these patterns is to use symbols in place of the words. Instead of the word In, we'll use a box to represent whatever number of coins we put into the pot. And instead of the word Out, we'll use a triangle to stand for the number of coins that comes out of the pot." Above the word *In* on the T-chart I drew a box, and above the word *Out* I drew a triangle.

□ In	△ Out
1	2
2	4
3	6
4	8
5	10
6	12
7	14
8	16
9	18
10	20
.	.
.	.
.	.
100	200

I pointed to the sentence that was Armando's idea: *The Out number is equal to the In number added to itself.* I asked, "What symbol did I suggest for the Out number?"

"A triangle," Rick said. I drew a triangle on the board under Armando's sentence, large enough for a number to be written inside it.

"What's the next part of Armando's sentence?" I asked.

"Is equal to," Audrey answered. I added an equal sign:

△ =

"Now what?" I asked.

"You write a box for the In number," Nina said.

"And then plus another box," Armando added. I did as they suggested:

△ = □ + □

"Triangle equals box plus box," I read. I pointed next to the sentence that was Annie's idea: *In plus In equals Out.* "Now let's use symbols to represent Annie's idea. How should I start?"

"Use a box for In," Beatriz said. I drew a box below the first *In* in the sentence.

"And then?" I asked.

"The plus sign," James said.

"I know what goes next," Cami said. "The next word is *In*, so I think a box goes next." Several students nodded. I added the plus sign and another box.

"You can use the equals sign for the word *equals*," Jaime said.

"What should I write after the equals sign?" I asked.

"A triangle because the Out column has the triangle symbol," David said.

I completed writing the equation:

In plus In equals Out.
$\square + \square = \triangle$

I said, "So this equation is 'Box plus box equals triangle.' Whose pattern does this equation describe?" I used the word *equation* without defining it, but referring to what I had just written. Using new terminology in the context of an activity is an effective way to introduce it.

"Nina's," Michael said. "It adds itself twice, and that's what Nina's pattern said to do."

"Let's think about Brianna's pattern that Sam described," I said, pointing to the board where I had written *The Out number is equal to the In number times two*.

"Brianna multiplies instead of adding," Andrea said.

"How can we write an equation using a box and a triangle for Brianna's pattern?" I asked, again using the word *equation*. Several hands went up, as students were eager to help.

"You use a triangle for the Out number," Kris said. I drew a triangle under the part of the sentence that said *The Out number*.

"Now what?" I asked the class.

"The equals sign comes next," Audrey suggested. I followed her suggestion.

"Then write a box for the In number and then 'times two,' " Jaime added. I finished writing the equation below the sentence:

The Out number is equal to the In number times two.
$\triangle = \square \times 2$

"Does anyone know what symbols like the box and triangle are called when they're used in equations like this?" I asked, pointing to the box and the triangle. No one seemed to know.

"They're called *variables*," I said, introducing the class to the correct terminology. "That's because the numbers they represent can vary. In this case, the number of coins that come out of the pot depends on the number of coins that go into the pot. You can put any number in the box and then you can figure out the number that belongs in the triangle." I ended class with this simple explanation of a very complex idea.

Day 2

In this part of the lesson, students build on the experience from the day before and explore what happens if the pot does something other than double what's put into it. To begin the lesson, I held up the book and pointed to the pot on the cover. I asked, "Who would like to tell what the pot does in the story?" Almost all hands were immediately in the air.

"It doubles," Beatriz said.

"It multiplies by two," Andrea added.

No one had any other ideas to add. Adam raised his hand. "I was absent and I don't know the story," he said.

I quickly retold the story of Mr. and Mrs. Haktak and the discovery of their magic doubling pot. I ended my retelling by showing the class the last few pages so that Adam could see how the pot had doubled everything, including Mr. and Mrs. Haktak.

I said, "Instead of doubling, today the pot is going to do something different. As Mr. and Mrs. Haktak did, you'll have to figure out what the pot is doing. We'll use a T-chart as a way of keeping track of the information to help us figure out the pot's rule." I quickly drew a T-chart on the board and labeled the left column *In* and the right column *Out*. As I had done the day before, I drew a box above *In* and a triangle above *Out*.

I continued with the directions, "I'll start by giving you a clue about what the pot is doing by writing on the T-chart what happens when one coin is put into the pot. Then I'll call on someone to guess the output value when two coins are put in. When I call on you, tell me what you think the Out number is, but don't say the rule for what the pot is doing. If you're right, I'll record your guess. If you're not right, then I'll record

□ **In**	△ **Out**
1	3
2	

the correct answer, and then you'll have more information for your next guess." The rule I had chosen was to multiply the number of coins put in by three. I wrote *1* and *3* on the T-chart, and then wrote a *2* underneath the 1.

"Raise your hand if you'd like to guess the next Out number." All students were eager and had raised a hand. I called on Karly.

"Four comes out," Karly said. Karly's incorrect guess was a typical one. Some children reason that I added two to the 1 to get the first output of 3, and they then add two to the new input of 2 to get 4. Others look at the 3 in the Out column and increase it by one to get 4, seeing that I had increased the 1 in the In column by one to enter the next number of 2.

"Your Out number makes sense, Karly, but it isn't the pot's rule," I

□ In	△ Out
1	3
2	6
3	

replied as I wrote *6* in the Out column. Some students were surprised, while others had their thinking confirmed. I wrote *3* in the In column.

"Who would like to guess the output value for three?" I asked. I called on Michael.

"Nine," Michael said. "Can I tell what it's doing?"

"No, not just yet," I said. "When four different students have made correct guesses, then someone can describe what the pot is doing. For now, you can just give output values for the input values. So far, that's one correct guess." Limiting the students to guessing outputs and not telling the rule before four correct guesses have been made provides a

□ In	△ Out
1	3
2	6
3	9
4	12
5	

way to keep the students involved and thinking. I recorded *9* in the Out column and then wrote *4* in the In column. Hands were waving. I called on Sam.

□ In	△ Out
1	3
2	6
3	9
4	12
5	15
6	18

"Twelve," he said with confidence. I nodded and recorded the *12*, and then a *5* in the In column.

"That makes two correct guesses," I said. Practically all hands were raised now and I called on Audrey. She gave the correct output value of 15 for 5, and then Brianna gave the correct output value of 18 for 6. Brianna's was the fourth correct answer.

"That's four correct guesses," I said. "Who would like to share what you think the pot's rule is?" Almost all hands were up, accompanied by pleading looks. I called on Adam.

"They go by threes," he said.

□ In	△ Out
1	3
2	6
3	9
4	12
5	15
6	18
7	21

"Which number are you talking about?" I asked.

"The Out numbers; they count by threes—three, six, nine, twelve, fifteen, eighteen," Adam said.

I wrote *7* in the In column and asked, "Can you figure out how many coins come out if I put seven coins into the pot?"

He nodded and said, "Twenty-one." I recorded *21* in the Out column.

It's typical for students to focus on the pattern of the numbers in the Out column. While this way of thinking is useful for predicting what comes next, it isn't practical for predicting output values when input values aren't offered in sequence. For example, figuring the output value for 25 or 37 calls for a different approach. However, this pattern is a starting place for students and it's important to value this thinking. I recorded Adam's idea on the board:

The Out numbers count by 3s.   Adam

"Does anyone have another way to describe what the pot is doing?" I asked. I called on Cami.

"The pot is multiplying what goes in by three," she explained.

"If we use your rule, would you get the same output value for seven as Adam did with his pattern?" I asked.

Cami replied, "Yes, seven times three is twenty-one. It works." In contrast to Adam's rule, Cami's rule is useful for figuring out the output value for any input value. I wrote a sentence starter on the board:

The Out number equals

I said to Cami, "Can you say your rule again using this sentence starter I wrote on the board?" I used this structure so that translating to equations would result in equations that followed the standard algebraic form, with the output variable first.

Cami said, "The Out number equals what goes in times three." I recorded on the board:

The Out number equals what goes in times three. *Cami*

"I know a shorter way from before to write it," Michael said.

"Share your idea with us, Michael," I said.

"Couldn't you just save time and write 'triangle equals box times three'?" Michael asked. He continued, "The triangle means the Out number and the box means the In number." I recorded on the board under Cami's statement:

$\triangle = \square \times 3$ *Michael*

"Mathematicians call this an *equation*," I reminded the class. "The box and the triangle are variables because the numbers they represent can vary, depending on the number of coins that are put into the pot. You can put any number in the box and then figure out the number that belongs in the triangle." I had planned to ask the children to shorten the description by writing an equation, and I was pleased that the suggestion came from Michael instead.

A Second Rule I said, "I'm now going to think of a new rule for the pot. Would you like it to be harder or easier?"

"Harder!" the class eagerly responded.

I left the first T-chart on the board and drew a new one beside it. I wrote *1* under the In column and *3* under the Out column. The rule I used was "The Out number equals two times the In number plus one."

□ In	△ Out
1	3
2	

There were several responses from the students. "It's the same as last time."

□ In	△ Out
1	3
2	5
3	

"It can't be, that would be too easy."

"She said she would make it hard!"

"Remember just to guess. If you think you know the rule, don't say it until it's time," I reminded the class as I wrote *2* in the In column. I called on Audrey.

"Four," she responded.

"That output value doesn't follow the pot's rule," I replied. I recorded *5* next to the 2 and then wrote a *3* in the In column.

Elisa said, "It really is a hard one."

"I'm confused," Audrey said.

"Talk this over with your neighbor and see if you can figure out what

□ **In**	△ **Out**
1	3
2	5
3	7
4	

the pot is doing," I suggested. After the students talked for a few moments, I asked for their attention.

"Who would like to guess for three?" I asked.

"Eight," Jaime guessed.

"No, that doesn't follow the rule," I responded. Other hands shot up to give a different answer, but I followed the procedure I had begun. I wrote the correct answer of *7*, giving the class more correct information, and then wrote the next input value of *4*. Now all hands were up.

I called on Elisa. "Nine," she said with confidence.

□ **In**	△ **Out**
1	3
2	5
3	7
4	9
5	11
6	13
7	15

"You're right. That's one correct guess," I said, recording the *9* and extending the In column to *5*. There were lots of comments from the others. "I get it now."

"It's easy."

□ In	△ Out
1	3
2	5
3	7
4	9
5	11
6	13
7	15
	.
	.
	.
	41

"I know the answer for five."

"I know the rule."

Brianna, Gary, and Andrea correctly guessed for the next three input values, bringing the total to four correct guesses.

"Before we talk about the rule," I said, "I want to give you an output value and see if you can guess the input value." I wrote *41* under the Out column, putting three dots after 15 to indicate that I was skipping some numbers.

The students thought for a few moments and then hands began to go up. Some students were writing on their papers. I know that some students are initially only comfortable following the pattern of the output numbers, looking down the numbers in the Out column. They haven't yet figured out how to get to an output value from an input value, or vice versa. At this time, some students were writing down the pattern of odd numbers to 39. Then they counted down to figure the corresponding input number. While this strategy works and is typical when students first explore patterns like these, it limits students' thinking for generalizing a pattern. However, with more experience, students learn how to identify output values from input values. When more than half the students had a hand raised, I called on Kris.

"I think the In number could be twenty," Kris said with uncertainty.

"That's what I got!" "Me, too!" were some of the responses of the other students. I wrote *20* under the In column and the class cheered. Kris looked especially pleased for answering correctly.

"What's the pot's rule?" I asked. "Remember to state the rule starting with 'The Out number equals.'"

"The Out number equals the In number plus the In number then plus one," Rick said.

I wrote on the board above the T-chart:

The Out number equals the In number plus the In number plus one.   Rick

"Let's try Rick's idea," I said. "If we use the In number of seven, then seven plus seven is fourteen, and fourteen plus one is fifteen. Is fifteen the output value for the input of seven?"

"Yes, it works!" James exclaimed, then apologized for blurting out. "Seven plus seven is fourteen and one more is fifteen."

"It works for six," Brianna added. "Six plus six equals twelve, plus one is thirteen. It checks."

"Can I tell a shorter way to write it?" Annie asked. I nodded. "Triangle equals box plus box plus one," she said.

I wrote on the board under Rick's sentence:

$\triangle = \square + \square + 1$ *Annie*

"I have a different idea," Nina said. "Instead of 'box plus box,' couldn't you write 'two times box'?"

I nodded and then said, "First, tell me in a sentence what you think the rule is. Use the sentence starter I wrote on the board."

Nina said, "The Out number equals two times the In number plus one." I wrote on the board above Rick's idea:

The Out number equals two times the In number plus one. Nina

"Is what I wrote what you were thinking?" I asked Nina. She nodded.

"I also know how to write it a shorter way," Nina added. "It would be 'triangle equals two times box plus one.' "

Under Nina's sentence I wrote the equation:

$\triangle = 2 \times \square + 1$

"Let's try Nina's idea and see if it works," I said. "Let's use the In number three and its Out number, seven. Is the Out number the box or the triangle?"

"Triangle," the students chorused.

I wrote the *7* in the triangle and the *3* in the box and read, " 'Seven equals two times three plus one.' Is that true?" I asked the class. "Thumbs up if you think it's true, thumbs down if you think it's false, and put your thumb sideways if you're not sure." All thumbs were up.

Rick explained, "Two times three is six, and then if you add one to six then that's seven, and that's what we're supposed to get."

"Can I try it with the Out number eleven for five?" David asked. "I think it works." I nodded and replaced the 7 and 3 in the triangle and box with *11* and *5*.

David said, "Eleven is the same as five times two, which is ten, and then add one and it makes eleven. See, it works!" There were no other comments.

I asked, "Does it surprise you that two different equations work to give the same numbers on the T-chart?" A few hands went up. "Talk with your neighbor about this," I said. After a few moments I asked for their attention.

Armando shared, "Sam and I talked and we don't think it's weird or anything that two rules work. We had two yesterday when the pot was doubling."

Cami said, "Multiplication is another way of writing addition. Nina just thought of Rick's way with multiplication instead of addition." Several others nodded.

I added, "Multiplication is another way of writing addition when all the groups are the same size or you have to add the same number several times."

A Third Rule "This time, I'd like to have one of you choose the rule for the pot," I said. I paused and gave the students a few moments to consider what rule they could suggest for the pot. I called on Andrea and asked her to come to the front of the room and whisper her rule into my ear.

She whispered, "Do two times the In number minus three." I agreed. I drew a third T-chart beside the first two, and wrote *1* in the In column.

"Do you know what the output value is for one?" I whispered back to Andrea.

She nodded and with a giggle whispered back, "Do negative one for the Out number." I mentally checked Andrea's arithmetic and wrote *–1* in the Out column. I've found that many children can figure this out without having had formal instruction about calculating with negative numbers. Those who couldn't, however, would still be able to figure out the rule from the other pairs of numbers on the T-chart.

□ In	△ Out
1	–1
2	1
3	3
4	5
5	7
6	9
7	11
8	13
9	15

I asked Andrea to return to her seat and I repeated the procedure of waiting for four correct guesses before asking someone to explain what the pot was doing. It took eight guesses after the clue for the students to correctly guess the Out number four times.

Jaime said, "Hey, the numbers are the same as the last T-chart, just pushed down."

"But my rule is different," Andrea said.

Michael then correctly stated the rule: "Andrea's rule is 'The Out number equals the In number plus the In number minus three.' " I wrote on the board above the T-chart for Andrea's rule:

The Out number equals the In number plus the In number minus three.
Michael

Sam pointed out that Michael's rule could also be written using multiplication. He said, "The Out number equals the In number times two minus three." I wrote on the board:

The Out number equals the In number times two minus three. *Sam*

"Would anyone like to write an equation for Michael's rule?" I asked.

"You can write 'triangle equals box plus box minus three,' " Audrey said. I asked Audrey to come to the board and write her equation under Michael's sentence. She wrote:

$\triangle = \square + \square - 3$

"What equation can we write for Sam's sentence?" I asked. Most hands were up quickly. I called on Cami, who suggested, "Triangle equals two times box minus three." She came to the board and wrote:

$\triangle = 2 \times \square - 3$

□ In	△ Out
1	$1\frac{1}{2}$
2	

□ In	△ Out
1	$1\frac{1}{2}$
2	2
3	$2\frac{1}{2}$
4	3
5	$3\frac{1}{2}$
6	4
7	$4\frac{1}{2}$
8	5
9	$5\frac{1}{2}$

A Fourth Rule "Who would like to pick the next rule for the pot?" I asked. I called on Kris. He came up and whispered his rule in my ear: "The Out number equals the In number divided by two plus one." When I drew the T-chart, Kris said, "The first clue is one and one and a half." I recorded these numbers and wrote the next input value of 2.

The fractions presented a complication for the students. However, some were able to use the increasing pattern of the output numbers to make correct guesses. After the input value of 9, they had made four correct guesses.

No one, however, was willing to state the rule. I asked, "Who notices how this T-chart is different from the others?"

"There are fractions," Brianna said.

Rick said, "Some of the Out numbers are smaller than the In numbers, and on most of the other charts, the Out numbers are bigger than the In numbers." There were no other comments.

"What operation causes whole numbers to get smaller?" I asked.

"Subtraction," James said.

"I agree," I responded. "What's another operation that causes whole numbers to get smaller?"

"Division," several students said. I nodded my agreement.

I said, "Here's a hint: Kris's rule has two steps. The first step is to divide by some number, and the second step is to add the number one. Talk with your neighbor. Use the information I gave you to figure what number Kris's rule divides by."

The discussions were animated as students grappled with the rule. When students thought they had a rule, I asked them to test it for the numbers on the T-chart. After a few minutes, I called for the students' attention.

"I think Kris's rule divides by two," Armando said. "I thought this because you can divide any even number by two with no leftovers. On the T-chart, none of the even numbers had leftovers."

Karly added, "If you divide an odd number by two, you could have a remainder of one or one-half. It depends on what you're dividing. If you're dividing cookies, then you could have one-half. If you're dividing bicycles, you'd have remainder one."

Brianna explained, "We got stuck on the 'plus one' part. We tried a bunch of different numbers on the T-chart and sometimes we added one and sometimes we subtracted one. If you start with the In number, then after you divide by two, you add one. If you start with the Out number, you subtract one and then multiply what you have left by two to get the In number."

"Who can state Kris's rule?" I asked. Most students were eager to do so. I called on James.

James said, "The Out number equals the In number divided by two plus one." I wrote this on the board above the T-chart. Annie came to the board and wrote the equation:

$$\triangle = \square \div 2 + 1$$

Annie's equation made sense mathematically, but it didn't follow standard algebraic form. I wanted both to honor Annie's contribution and to introduce the students to another way to write the equation. I said, with a light touch, "And here's another way to write Annie's equation." I wrote on the board:

$$\triangle = \frac{\square}{2} + 1$$

I explained, "In this form, it's as if I've replaced the two dots in the division sign with the box and the two. It's another mathematical shortcut, and you'll usually see algebra equations written like this when division is involved." These students hadn't studied fractions in depth and most weren't comfortable yet seeing how division and fractions related, so I left my comment at that. Time was up, and I ended the class.

Day 3

I began the lesson by asking the class, "What are some other rules the pot might follow?" My plan was to have students work in pairs to choose a rule, write it as an equation, and generate a T-chart for it. A difficulty that sometimes arises when students create their own rules is that they make the rules too hard. By brainstorming a list, I have some control over the difficulty of the rules. Also, creating a list gives support to students who might have difficulty deciding on a rule on their own.

Cami suggested, "The pot could double and then subtract five."

Sam said, "It could multiply whatever goes in by four and subtract two."

"My idea is a little different," Armando said. "Maybe the pot could add fifteen and subtract three."

As the students made their suggestions, I listed them on the board.

Double and subtract 5.
Multiply by 4 and subtract 2.
Add 15 and subtract 3.

"So far all of you have suggested rules that take two steps. Could the pot do just one thing?" I wanted to make sure that there were some simpler rules for students to choose from.

Michael nodded and said, "Maybe the pot just multiplies everything by ten."

"Maybe it could just add nine every time," Annie said.

"The pot could divide everything that goes in by two," David said.

"Or maybe it could divide everything by two and then multiply by two," Brianna said. I added these ideas to the list:

Multiply by 10.
Add 9.
Divide by 2.
Divide by 2 and then multiply by 2.

When I had a dozen ideas listed on the board, I gave directions to the class: "You're going to work with the person sitting beside you and together choose a rule. You may pick a rule from the list on the board or you may make up one of your own. Be careful that you can figure out the In and Out numbers for the rule you choose. On the back of a sheet of paper, write your names and also write the rule as an equation. Then, on the front of the sheet, draw a T-chart. Label the columns as I've done and write seven or eight pairs of numbers on it."

I had given quite a few directions, so I asked the class, "Who can tell me what to do first?"

David said, "Choose a rule. We can get it from the board or make it up."

Brianna continued, "Get a sheet of paper and write our names and our rule."

"Where do you write your names and your rule?" I asked.

"On the back," the class replied in unison.

Beatriz said next, "Make a T-chart using your rule on the plain paper."

"How many pairs of numbers should you have on your T-chart?" I asked.

"Seven or eight," Nina said. I wrote the directions on the board as the students gave them.

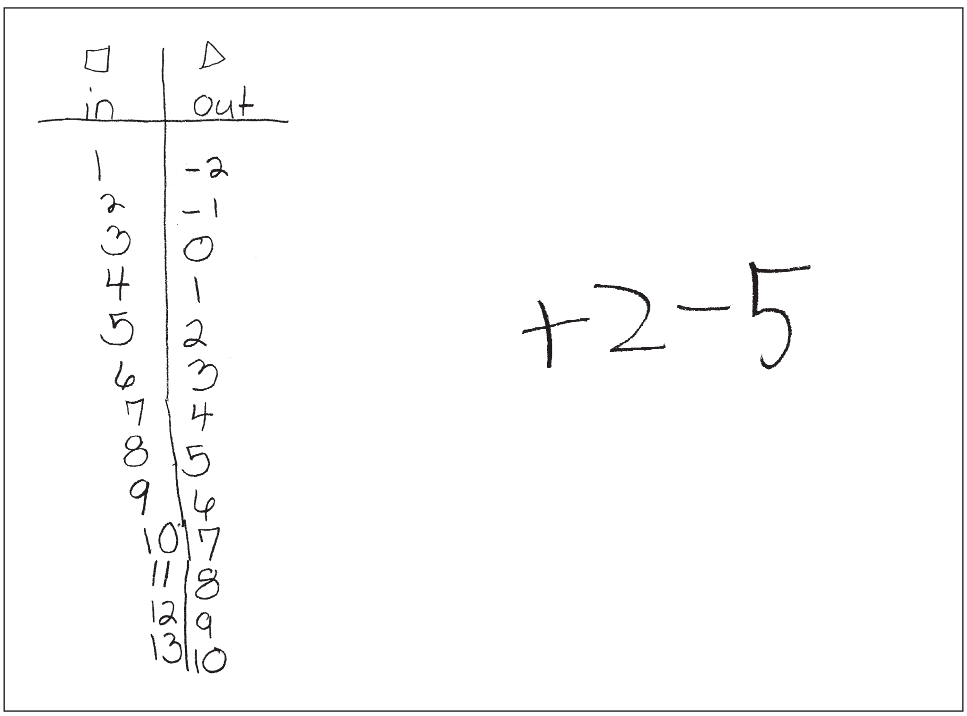
1. *Choose a rule from the board or make it up.*
2. *Write your names and rule on the back of a sheet of paper.*
3. *Make a T-chart on the front of the paper with 7 or 8 pairs of numbers.*

There were no questions and the students got to work. Most chose rules from the board, but some made up their own. Elisa and Michael chose the rule from the board of adding nine to the In number. Brianna and David made up their own rule of adding five to the In number and then subtracting three. These two pairs worked quickly and were done before many of the other students. I asked each pair to write their T-chart on an overhead transparency.

As I was circulating and observing the students at work, Kris and Annie called me over. Their rule was to add two and subtract five. "We're stuck," Annie said. "We figured out the Out number for three. Three plus two equals five and then subtract five and that's zero. But we can't figure out the Out number for one."

Kris added, "If we do the rule to two, it's two plus two, which is four.

FIGURE 1–1 Kris and Annie's rule resulted in negative Out numbers for the In numbers 1 and 2.



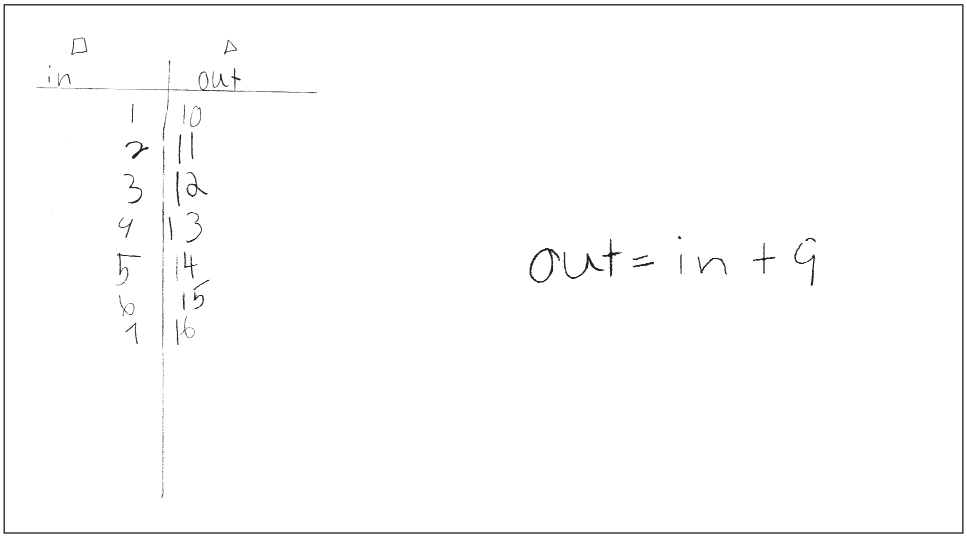
But then when we subtract five, we run out of numbers."

"Can we use negative numbers?" Annie asked. I nodded.

"That's easy, then," Kris said, looking relieved. "The Out number for two would be negative one."

"And the Out number for one would be negative two," Annie said. They completed their T-chart. (See Figure 1–1.)

FIGURE 1–2 Elisa and Michael's rule was *Out* = *In* + 9.



Most of the students were close to finishing and I gave a one-minute signal. Then I asked for the students' attention.

A Class Discussion To begin a discussion, I said, "I asked Brianna and David and Elisa and Michael to write their T-charts on overhead transparencies. Let's start with Elisa and Michael's." I projected their T-chart and said to the class, "Take a moment to study their T-chart quietly." (See Figure 1–2.)

When a moment had passed I added, "Talk with your neighbor about what you think the rule is and why. Be sure both of you have the opportunity to talk." After a few more moments, I called for the students' attention once again. The students were eager to share. The rule Elisa and Michael chose was simple for the others to figure out: "The Out number equals the In number plus nine."

I asked, "Using the sentence starter 'The Out number equals,' who would like to share what you think the rule is?"

Beatriz volunteered, "The Out number equals the In number plus nine." The rest of the students showed their agreement with thumbs up.

"Who knows what the equation could be?" I asked.

Gary said, "Triangle equals box plus nine." I recorded on the board:

△ = □ + 9

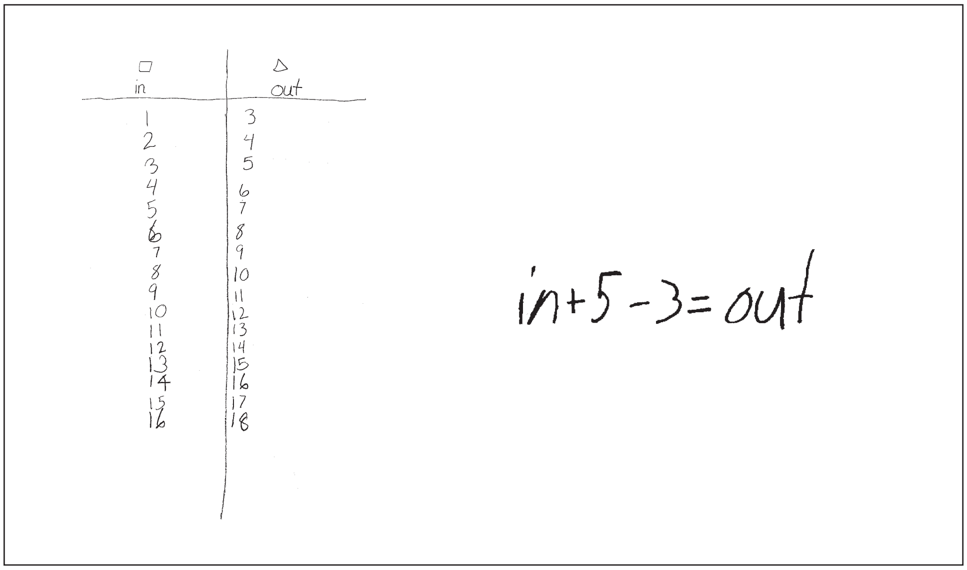
We checked the rule for several pairs of numbers from the T-chart, writing the numbers in the triangle and the box and doing the arithmetic mentally.

Then I projected Brianna and David's T-chart. (See Figure 1–3.) I repeated the process I had used with Elisa and Michael's T-chart, asking students first to study the T-chart by themselves and then to talk with their partners. Many students were interested in telling the rule. I called on Sam.

"The Out number equals the In number plus two," Sam said. "It's 'triangle equals box plus two.' " Most of the others showed their agreement with thumbs up.

"That's not it," David said, with a sly smile. The others were shocked.

FIGURE 1–3 Brianna and David's rule was *In + 5 – 3 = Out.*



"It has to be!" James protested.

"No, it's not right," Brianna insisted. Conversation broke out across the room. I called the class to order.

"Tell me again what you think the equation is, Sam, so I can write it on the board," I said. Sam repeated his idea and I recorded:

$\triangle = \square + 2$

Nina said, "But it works for all of the numbers on the chart."

"I agree," I responded, "but I also know that Brianna and David have a different rule in mind that also works for all of the pairs of inputs and outputs on the chart." The class was stumped.

"How about we give them a hint?" I asked Brianna and David. They agreed.

I said to the class, "Brianna and David have a two-step rule." The class remained silent. Then Jaime's hand shot up.

"I know," he said. "They added one and then they added one again. It would be 'triangle equals box plus one plus one.' " Brianna and David giggled and shook their heads "no." I recorded on the board below Sam's idea:

$\triangle = \square + 1 + 1$

The class was quiet again, so I gave another hint. "They used addition and subtraction in their rule," I said. Audrey wanted to make a guess.

"Is it 'triangle equals box plus four minus two'?" she asked. Brianna and David again giggled and shook their heads as I recorded below Jaime's idea:

$\triangle = \square + 4 - 2$

"Tell the class your rule," I then said to Brianna and David. As they reported, I recorded their rule underneath the others:

$\triangle = \square + 2$
$\triangle = \square + 1 + 1$
$\triangle = \square + 4 - 2$
$\triangle = \square + 5 - 3$

"That's not fair," Rick said. "It's just another way of writing the first one."

"Why do you say that?" I asked.

"Because five minus three is two," Rick said. "It's just another way to write two."

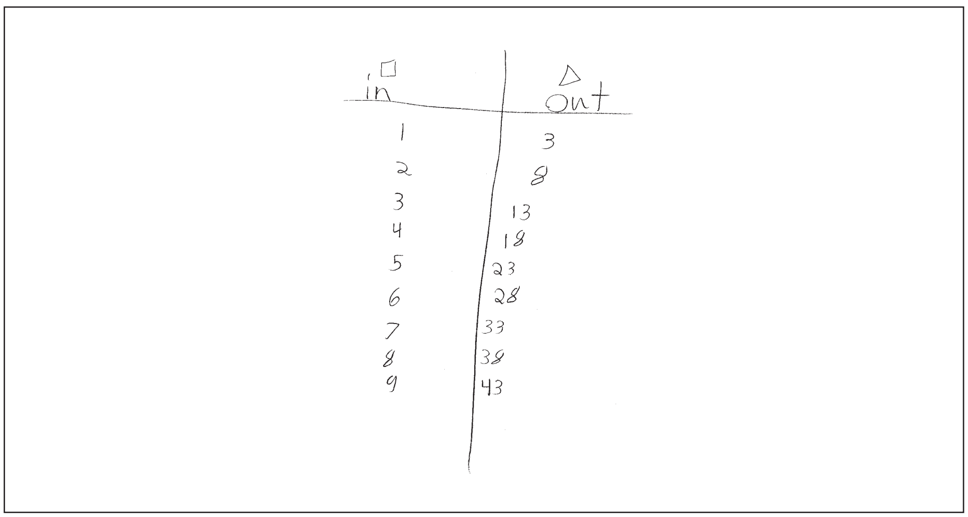
"So is four minus two," Audrey added.

"All of these equations work for Brianna and David's T-chart," I said. "They all produce the same output number for an input number. Equations like these are called *equivalent equations*. Mathematically, they're all correct, but they are different ways to describe the T-chart. Your job is to find any equation that works."

For the rest of the class, other pairs of students came up and recorded their T-charts on transparencies. As we did for the others, first the other students thought quietly, then they talked with their partners, and finally I called on someone to tell a rule and then someone else to tell an equation.

Time ran out before all of the students could present their rules, and some were disappointed. "You'll all have a chance to do so on other days," I reassured the students.

FIGURE 1–4 Rick and Audrey's rule was *In* × *5* – *2* = *Out.*



"Can we do a different rule?" Audrey asked.

I responded, "Yes, if you'd like you can do another, but you won't be able to share it until everyone has had a chance to share one rule."

I collected their papers and saved them for a later activity, when the students would make graphs of their patterns. Over the next few days, the rest of the students had the chance to present their T-charts for the others to guess their rules. Figure 1–4 shows one more pair's T-chart.