Algebra Zapping Zombies

A Lesson for Grades 6–8

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The lesson was adapted from the Math Solutions publications It's All Connected: The Power of Proportional Reasoning to Understand Mathematics Concepts, Grades 6–8.

Overview of Lesson

In this lesson students translate among representations so that their understanding can advance from words to tables to graphs to equations. The concepts are covered in the context of playing a video game where Zombies are zapped to score points and advance a level in the game. Introduce to students the background about the video game. The main goal of the game is to earn points by zapping Zombies. Have a brief conversation about game levels and points earned. As students work through this lesson keep them focused on the relationship between the different parts of the table.

The understanding of proportional relationships in this problem helps students make the connections between the different representations.

Common Core State Standards

Expressions and Equations 7.EE

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

NCTM Correlation

Number and Operations

Compute fluently and make reasonable estimates.

Develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.

Algebra

Use mathematical models to represent and understand quantitative relationships.

Model and solve contextualized problems using various representations, such as graphs, tables and equations.

Analyze change in various contexts.

Use graphs to analyze the nature of changes in quantities in linear relationships.

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Goals for Student Understanding

- Represent relationships in a tabular, verbal, graphical, and symbolic form.
- Compare table and graphic representations of the same data and connect to the symbolic representation.
- Identify and understand how unit rate is represented in an equation, table, and graph.

Materials

- Paper
- Pencil
- Graphing calculator (optional)
- Graph paper (optional)

Key Vocabulary

Ratio

Rate

Unit Rate

Slope

Constant rate

Coefficient

Independent variable

Dependent variable

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Teaching Connections

- Make sure students can read the table by reviewing the information known and ask questions about the information needed to complete the table. Leave the discussion open enough to allow students to decide on a method for finding the missing values.
- To help struggling students, suggest students first write a verbal description of the situation.
- Before discussing rates, review the definition of a ratio.

  A **ratio** is a comparison of two quantities. Ratios can be written using three formats.

  2 to 3; 2:3, and 2/3.
- Use guiding questions to help students understand the significance of what a unit rate is and what role it plays in generalizing the pattern. They then can translate their rule to symbolic form using variables.

  A **rate** is a ratio of two measurements.

  A **unit rate** is a rate where one of the measurements is 1. Sample unit rates are: 12 inches:1 foot, $25 per ticket, 4 hours:1 room
- Have a discussion to review the components of a graph in preparation for students graphing relationships in this lesson. Include in the review the concept of slope.

  Slope is the constant rate of change of a linear relationship. When the equation of line is given in a form

  $y = mx + b$, the coefficient of the x-term, m, is the slope .
- Instruct students on how to distinguish between the independent and dependent variables in a relationship.

  An **independent variable** in a relationship, or ordered pair, is the variable that stands alone and is not affected or changed by the relationship.

  A **dependent variable** in a relationship, or ordered pair, is the variable affected by the independent variable. The value of the independent variable changes the value of the dependent variable.
- Explain that equations also show how variables are related to each other, just like tables and graphs.
- Encourage students to share their strategies for completing the table.
- For students that use a repeated addition process to fill in parts of the table, ask questions that will focus them on the multiplicative nature of the values.
  - As the level numbers tripled from 2 to 6, how did the number of Zombies change? How did the number of points change?
  - How would you determine the number of Zombies you zapped if you reached level 20? Level 5?

Student Connections

Students need to complete the Student Recording Page. As students begin working ask them to estimate the values that belong in the table and explain why their estimates make sense. As students begin graphing, be sure they have identified the independent and dependent variables correctly and have selected to use an appropriate scale. As students work on this recording page, continue to ask questions about the relationships between variables of the situations.

Questions for Struggling Student

- What information variables are involved? *Levels, Zombies, and Points*
- What information can you gather from the table? *Possible answers include:*

  When you reach Level 6 you have zapped 210 Zombies and earned 23,100 points.

  You earned 7,700 points for Level 2.

  To exit Level 8 you need 30,800 points.

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- What information in the table will help you find the number of Zombies needed to advance to the next level? *The number of Zombies increased from 140 to 210 between Levels 4 and 6; therefore, the number of Zombies needed to complete two levels of play is 70.*
- What rate will help you complete the table? *210 Zombies zapped to 23,100 points earned. The unit rate for this ratio is 1/110, so I can use that to find other amounts of points earned.*
- Describe any patterns you see. *Answers will vary.*
- How are the level numbers changing in the table? *by 2 each time*
- How are the numbers of Zombies zapped changing in the table? *by 70 for every 2 levels*
- How do the numbers of points earned change in the table? *by 7,700 every 2 levels*
- You know the number of Zombies zapped for every two levels of play. Use that information to find how many Zombies need to be zapped at every level? This is called the unit rate. What other unit rates can you find? *For every level I need to zap 35 Zombies. 1 Zombie gets me 110 points, every level I earn 3,850 points.*
- Identify a ratio found in the table. the ratio between the level and the number of Zombies zapped; *the ratio between the numbers of Zombies zapped to the number of points earned; or the level to the number of points earned.*
- What do you know about Level 10? *the number of Zombies zapped;.* Given that, what can you determine about Level 5? *350 Zombies zapped, 38,500 points earned; Level 5 should be half of each of the values.*
- What is the relationship between the levels earned and the numbers of Zombies zapped? *For every two levels you increase the number of Zombies zapped by 70.*
- What labels do you use for the graphs? *Levels and Zombies, Zombies and Points, or Level and Points*
- For each variable, name the minimum and maximum values you need to include on your x- and y-axes. What is a reasonable scale to use for the axes? (e.g. What would be an effective way to skip-count, if your range went from 0 to nearly 50,000?) *for Levels, a scale of 1 with a minimum of 0 and a maximum of 15; for Zombies, a scale of 35 with a minimum of 0 and a maximum of 500; for Points, a scale of 5,000 with a minimum of 0 and a maximum of 50,000*

Questions for Students on Task

- Describe how you find the missing values to complete the table. Show examples to explain your thinking. *Use the values you know to help find the missing values. Find out how the table changes from one value to the next both horizontally and vertically.*
- What patterns do you see in the table? Students may suggest:

  As levels increase by 2, the number of Zombies increases by 70.

  As levels increase by 1, the number of Zombies increases by 35.

  As levels increase by 1, the number of points increases by 3,850.

  Ask students questions about any patterns that are not mentioned.
- If the pattern shows that the number of Zombies zapped increases by 70 Zombies for every two levels, how many Zombies need to be zapped per level? 35
- Describe the relationships between the variables. Press students to be as specific as possible. Students may suggest the following:

  As the levels increase, the other variables increase. Suggest student to be more specific by asking “By how much?”

  For every two levels you advance, the number of Zombies zapped increases by 70.

  For every 70 Zombies, you score 7,700 points.

  For every level increased, you zap 35 Zombies.

  For every level increased, you earn 3,850 points.

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- What are the attributes of a proportional relationship? *As the independent variable increases by one unit, the dependent variable increases at a constant rate. If you look at a graph, you see a linear relationship that passes through the origin.*
- What relationships can be graphed? What do you predict the graphs to look like? *Level and Number of Zombies Zapped; Level and the Number of Points earned; and Zombies Zapped and Number of Points earned; The graphs will be linear.*
- How can you tell if a relationship is proportional using a graph? *The graph is linear and passes through the origin.*

Questions for Students on Task

- Suppose Leo purchased a newer version of Zapping Zombies. In this version, the number of Zombies zapped at each level increases from 35 to 50 Zombies. How would this affect the original table of values? How would this affect the original graph? How would this affect the original equation? *In the table, the values would increase more rapidly; in the graph the trend line would be steeper; and in the equation the coefficient of x would be 50, not 35.*
- What would the values be if you extended the table to level 13? *The number of Zombies zapped would reach 455; the number of points earned would be 50,050.*
- After playing for many hours, Leo had a total of 92,400 points. What level did he complete? Show how to use the table, graph, and equation to justify your solution. *He would be on level 24. I knew that each level was worth 3,850 points, so I divided 92,400 by 3,850 to get a quotient of 24. I could follow the trend on the graph to estimate the level and use my equation* p = 350l *to solve for* l *where* l, *is the level.*
- Leo's little brother said he beat the level and zapped a total of 990 Zombies. Leo didn't believe him. Mathematically justify which brother is correct. *Level 24 would only zap 840 Zombies; 24 • 35 = 840 Zombies zapped. Leo's little bother is incorrect.*
- If the bonus round scores triple points for every Zombie zapped, and you earn 16,500 points in that round, how many Zombies did you zap? Explain. *You normally earn 110 points per Zombie; if points are tripled, then you earn 330 points per Zombie. 16,500 ÷ 330 = 50 Zombies.*
- What benefits does the table, graph, or equation give you in understanding the specific relationships? *The table shows you specific data points. The graph shows you the relationship holistically, such as what is happening with the relationships. The equations tell you about the specific rates in the relationships.*

Lesson Summary

Students will experience identifying patterns involving a constant rate of change, completing a table, creating graphs, and writing equations. Students should recognize that all of these representations show that the relationships are proportional. They all increase with a constant rate of change as the other variable increases by 1. The relationships are all based on the proportional relationship between the levels, Zombies, and points. Have students go back to the fact that all of these relationships are balanced the same way in each representation. They all share a constant of proportionality among the compared variables.

Continue to focus students on the patterns of change (i.e. unit rate) in each equation. Discuss the connection between the coefficient of the variable in the equation, which is also the slope of the line, and the rates in the table. As students move through the topic of algebraic reasoning, it would be helpful for students to recognize how the different representations are related and how each representation is equally valuable.

Students should be able to tell you that the information that is readily seen from the table is the exact values of the ordered pairs. Information that is readily seen from the graph is the general pattern of the relationship. From the equation, you can find solution, even those can be efficiently determined without drawing a table or graph.

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Check for Success

- For each relationship, students should be able to identify the constant of proportionality. Generate a list of all the patterns observed (horizontally and vertically) in the table. Have students share their strategies for finding the missing values in the table. Record these strategies and have students record at least one strategy they fully understand. Focus students on the multiplicative relationships.
- Students should be able to generate a list of all possible unit rates and explain the information that can be gathered from a unit rate.
- Have students share their graphs so that they can compare and contrast the graphs. Have a class discussion about the following: all relationships are linear, all graphs of the line pass through the origin, proportional relationships are linear, but can have different rates. Include the following questions in the discussion so that students can verbalize what they notice in the graphs and what they have learned. Did you expect the graphs to look like this? Why?

  Did students use the same variables? Scales? Axes?

  What intervals are used?

  Are the relationships linear? How do you know?

  What other information can you gather from the graph?

  Can you make predictions about higher levels in the game?
- Have students summarize and explain their equations. You can check their understanding by posing any of the following questions.

  What steps did you use to write the equations?

  Is it helpful to write a rule in words before you write the equation?

  How can you write a rule as an equation?

  In words, what does each equation mean?

  What variables did you choose for your equation? Why? Can you define each variable?
- Suggest the following scenario with students. Tell students that Leo purchased *Beyond Zapping Zombies.* When looking at the game codes on line, Leo found the table of data given below. Have students predict how the tables, graphs, and equations will differ.

Level	3	6	9	12	15	18
Number of Zombies Zapped	180		540		900	
Points Earned		27,000		54,000		81,000

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Student_____Class_____Date_____

Leo likes playing a video game where he has to zap Zombies and turns them into statues before they invade a town. In the game, Zombies hide everywhere. Leo's goal is to clear all the Zombies to make the town safe again.

To advance to the next level you have to zap all the Zombies in that level. Each level has the same number of Zombies and you get the same number of points for each zapped Zombie. As you move through the levels, the Zombies get harder to zap.
Leo made the table below to show a relationship between the number of Zombies zapped and points earned.

Level	2	4	6	8	10	12
Maximum number of Zombies zapped		140	210		350	
Points earned	7,700		23,100	30,800		

1. Complete the table above. Look for relationships and patterns in the table. Describe at least three patterns.

2. Describe the ratios between the levels and the maximum number of Zombies zapped. How are these ratios related?

3. What are the ratios of the number of Zombies zapped to the number of points earned? How are these ratios related?

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4. Name at least two different rates you can find in this game.

5. List the unit rates in this problem? How do you know the rates are unit rates?

6. Graph each rate. How do they support your findings from Question 5?

7. Write an equation for each relationship described below.

   a. the level in the game and the number of Zombies zapped

   b. the number of Zombies zapped and the points earned

   c. the level in the game and the points earned

8. How did you use a unit rate to write each equation?

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Student Recording Page Answer Key

1.

Level	2	4	6	8	10	12
Number of Zombies Zapped	70	140	210	280	350	420
Points Earned	7,700	15,400	23,100	30,800	38,500	46,200

1. Levels increase by 2.
2. Zombies zapped increases by 70 for every 2 levels.
3. Points earned increases by 7,700 for every 70 Zombies zapped.
4. Each level you earn 3,850 for every 35 Zombies zapped.
5. You can earn 3,850 points in every level for zapping Zombies.

2. Answers may include:
   - There is a ratio of 2 levels: 70 zapped Zombies.
   - There is a ratio of 6 levels: 210 zapped Zombies.
   - There is a ratio of 12 levels: 420 zapped Zombies.
   - There is a ratio of 1 level: 35 zapped Zombies.
   - There are 35 Zombies per level.
   - All ratios are equivalent. $\frac{2}{70} = \frac{6}{210} = \frac{12}{420} = \frac{1}{35}$

3. Answers may include:
   - There is a ratio of 70 Zombies zapped: 7,700 points earned.
   - There is a ratio of 210 Zombies zapped: 23,100 points earned.
   - There is a ratio of 420 Zombies zapped: 46,200 points earned.
   - There is a ratio of 35 Zombies zapped: 3,850 points earned.
   - There is a ratio of 1 Zombie zapped: 110 points earned.
   - For each level the ratio of the number of Zombies zapped compared to the number of points earned is equivalent.
   - Each zapped Zombie is worth 110 points.

4. Answers may include:
   - There is a rate between the levels and Zombies zapped.
   - There is a rate between Zombies zapped and points earned.
   - There is a rate between levels and points earned.

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5. Answers may include:

I believe all the rates in the table are constant. All numbers are increasing by a constant factor. As the levels increase by two the number of Zombies zapped increases by 70, and the number of points increases by 7,700.

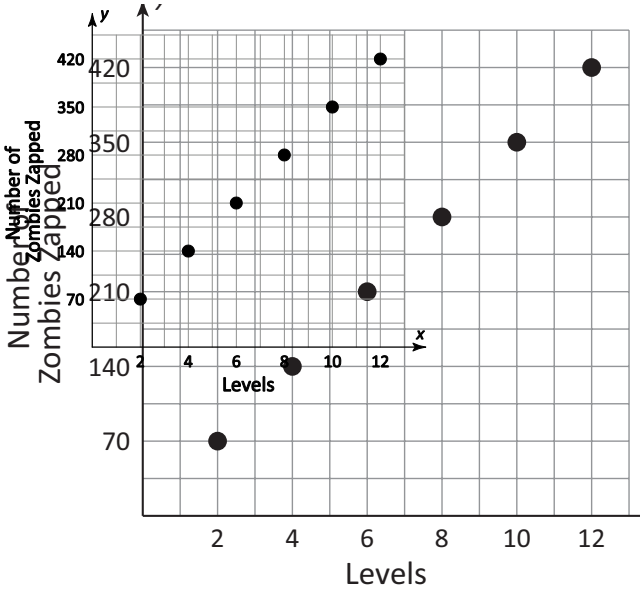
Levels to Zombies: $\frac{2 \text{ Levels} \times \frac{1}{2}}{70 \text{ Zombies} \times \frac{1}{2}} = \frac{1 \text{ Level}}{35 \text{ Zombies}} = \frac{2}{70} = \frac{1}{35}$

Zombies to Points: $\frac{70 \text{ Zombies} \times \frac{1}{70}}{7{,}700 \text{ Points} \times \frac{1}{70}} = \frac{1 \text{ Zombie}}{110 \text{ Points}} = \frac{70}{7{,}700} = \frac{1}{110}$

Levels to Points: $\frac{2 \text{ Levels} \times \frac{1}{2}}{7{,}700 \text{ Points} \times \frac{1}{2}} = \frac{1 \text{ Level}}{3{,}850 \text{ Points}} = \frac{2}{7{,}700} = \frac{1}{3{,}850}$

These rates can also be described as follows: for every level you have to zap 35 Zombies; for every Zombie zapped you score 110 points; and for every level you complete you score 3,850 points.

6.



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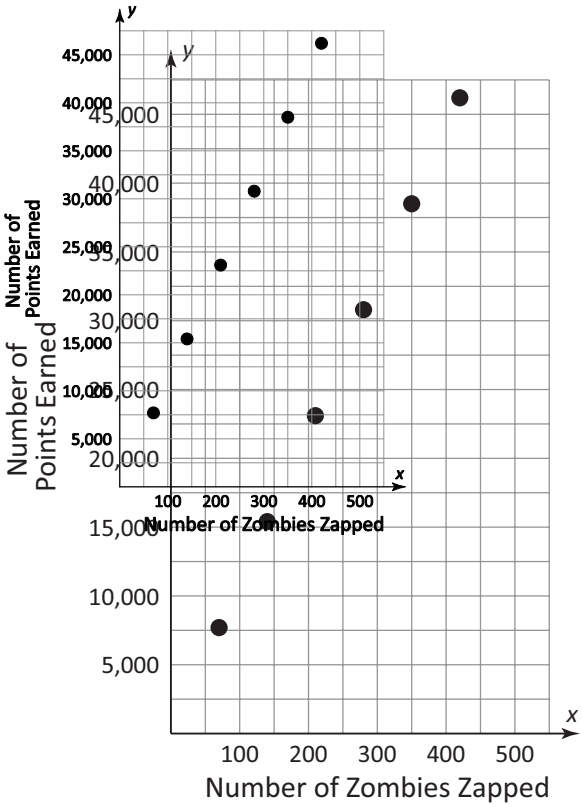

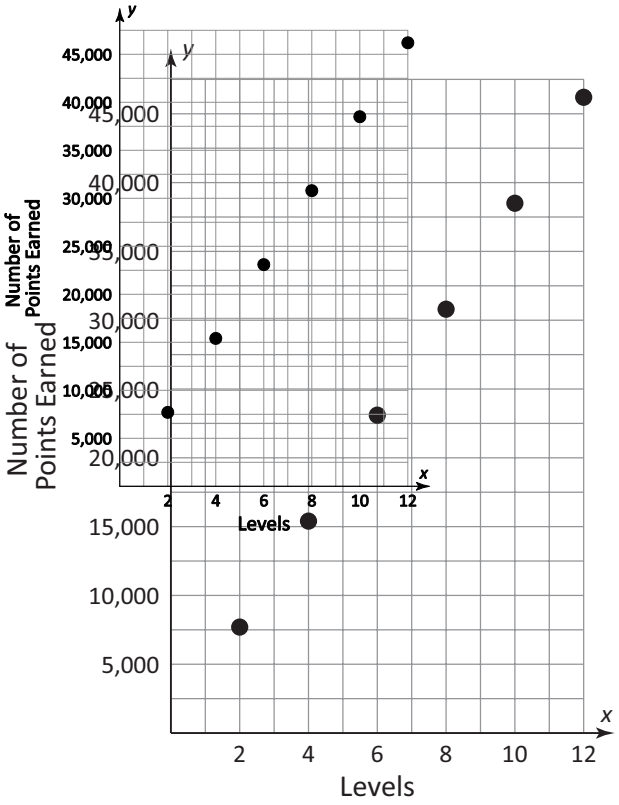

7. a. $z = 35l$; b. $p = 110z$; c. $p = 3850l$

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8. Answers may include:

The table helped me. I found the constant rate of change which became the coefficient of the *independent variable* in the equation.

From the table I saw the unit rate for each relationship. If you know the amount of change per variable, then you can multiply that value by any number (or independent quantity) to find the total dependent quantity.

From the graph I saw that it was a linear relationship and then found the slope (the amount of vertical change for every one unit of horizontal change). Finally, I wrote the equation y = slope times x.

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