

How to Use This Resource

Overview

It's All Connected was created in the belief that concentrating on graphs can be a highly effective approach to mastering algebraic concepts. The 40 lessons, which include reproducible student pages, ease students and teachers into algebra through the study of the graphical representation of functions. Each lesson seamlessly supports core curriculum.

The short lessons in this resource (each takes approximately fifteen to twenty-five minutes) are ideal for implementation with students in the classroom, and they can also be used by teachers for the purpose of professional development. For classroom use, these mini-lessons should be taught one to three times a week throughout the academic year.

Why This Resource?

Research reveals that many students struggle with or never develop the ability to easily move from one representation for a function to another. Because this is a necessary skill for mathematical understanding, these students are at risk for failure in mathematics. Research also reveals that prospective teachers have especially low scores on assessments that evaluate flexibility between visual and algebraic representations and flexibility with real-life situations. This resource provides the support needed to significantly improve both students' and teachers' algebraic understanding.

Learning beginning algebra is much enhanced by the early introduction of graphs and indeed can be first approached using graphs. There is a natural progression from qualitative graphs (graphs without scale), to quantitative graphs (graphs with scale), to tables, and finally to equations. By describing relationships between familiar real-world quantities, the graphs in this resource support students (and teachers) in perceiving algebra as practical and concrete from the start.

"Studies show that flexibility with visual representations is the most important factor in predicting [teachers'] content knowledge. This suggests that it would be beneficial for teacher educators to provide prospective elementary and middle school teachers more opportunities to solve problems with graphical representations of linear functions."

—*Journal of Mathematics Education*,
June 2010

WHAT IS THE PHILOSOPHY OF THIS RESOURCE?

Over the years, mathematics teachers have been urged to make profound changes in content, teaching style, and assessment. The traditional approach of presenting a rule, giving a few examples, and then working a series of exercises based on the examples is not seen as the optimal method of promoting true learning. The current philosophy is that the emphasis should be on student thinking, problem solving, and comprehension of underlying concepts. Moreover, applications should play a prominent role. Students need to be given opportunities to formulate problems, reason mathematically, make connections between mathematical ideas, and write about mathematics. The

focus is away from symbolic manipulation and courses that concentrate on developing a set of procedural skills.

This resource naturally aligns itself with the current philosophy in mathematics education. Understanding the graphical representations of functions cannot be achieved in a procedural fashion. Applications are involved in every lesson, and students are consistently asked to formulate their own problems. Interpreting graphs requires students to think on an abstract level.

Changing outlook, content, and teaching style is not an easy process. It tests and taxes the creativity, commitment, time, adaptability, and energy of teachers, departments, and administrators alike. A related challenge is the prevalence of high-stakes testing and the pressure teachers feel to “teach to the test.” The combination of change and pressure is particularly hard on middle school teachers who often have to teach more than one subject area and are not required to be certified in mathematics. It is the goal of this work to make the graphical representation of functions not only accessible to students and teachers but also an exciting new domain that they will want to explore and incorporate fully into their base of mathematical knowledge.

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Thinking Abstractly

In each of the forty mini-lessons, the initial effort to picture a relationship prompts students to think on an abstract level. This is an excellent way for students to understand the concept of a function—seeing how one quantity relates to another. Students first recognize the appropriate graph to use and then move on to creating their own examples. This approach allows students to perform the abstract process of representation and analysis with graphs before having to deal with algebraic notation. Producing their own graphs naturally assimilates students to the mathematics standards of communication and representation.

Analyzing Change

Analyzing change is another major component of this work. The lessons begin by examining change that is constant and then introduce the notion that change can be represented as occurring increasingly faster or slower. This is a key concept in calculus and should be introduced in middle school by looking at and interpreting the shapes of curves. Both researchers and teachers have noted that students often fail to see that variables vary. Students tend to see x as some specific (albeit perhaps unknown) quantity. A graph emphasizes the fact that x varies, taking on different values with movement along the horizontal axis.

Moving from Graphs to Equations

Throughout the middle school and high school mathematics curricula, students struggle with the concept of underlying equivalence. Changes in form and representation often cause students to lose sight of the fact that they are producing something equivalent to what they started with. This is particularly true when students move from an equation to a graph. Because a graph and an equation look so different, students do not grasp the fact that they are two representations of the same set of points. In the research literature, this problem is referred to as “not seeing the Cartesian connection.” The use of a

graphics calculator can make graphs far more accessible but does not necessarily help students see the Cartesian connection. The fact that every point on the graph satisfies the equation and that every solution to the equation is a point on the graph often comes as a surprise to students and takes them a long time to assimilate.

Teachers are urged to emphasize and reiterate often the fundamental relationship between graphs and equations.

The Importance of the Graphical Representation of Functions

Before the advent of graphics calculators, graphs played a much smaller role in the high school mathematics curriculum. Graphs were, for the most part, imperfectly produced by students, and the exercise to make a graph was the end, in and of itself. With the introduction of calculators that produce graphs, the graphical representation of a function can be as useful as and sometimes more useful than the equation corresponding to the function. The graph provides an overall picture of the relationship and indicates intervals of change.

The increase in attention to the graphical representation of functions makes it important for students to come into high school comfortable using graphs and able to understand the fundamental principles associated with them. Because this emphasis is relatively new, not a great deal of material about it is available for use by middle school teachers. This resource book provides that material. By using it in the classroom, teachers can help students thoroughly understand the fundamental principles of graphs and the underlying connections between graphs and equations.

How Is This Resource Organized?

It's All Connected has been carefully developed and organized to make it helpful for and easy to use by teachers. Familiarity with the graphical representation of functions is not assumed.

As outlined in the Contents, this resource has forty minilessons presented in four sections:

- Section I: eleven (11) lessons on qualitative graphs (picturing relationships students are familiar with)
- Section II: twelve (12) lessons on quantitative graphs (emphasizing units of measurement)
- Section III: eight (8) lessons on using tables with graphs
- Section IV: nine (9) lessons on equations (naturally arising from the data and graphs)

There are more lessons on graphs than on tables/equations because graphs are the subject proven to be most unfamiliar to teachers.

Each lesson takes between fifteen and twenty-five minutes to complete and contains two main parts: Teaching Suggestions and a Reproducible. The Teaching Suggestions section is divided into the following subsections:

Calculators as “Transformer” Toys

I worked with a ninth-grade teacher several years ago, when so-called Transformer toys were popular. With these toys, one could start with a robot and, through a few specific twists, produce a boat or some other totally unrelated object. It occurred to the teacher and me that students were looking at calculators like Transformer toys: One fed in an equation and out came a graph, but there was no real relationship between the two.

OVERVIEW

The overview summarizes the content of the lesson. It specifies the concepts to be introduced or reviewed and indicates the tasks that students will be required to perform.

CLASS TIME

Duration of lesson is noted; the times vary between 15 and 25 minutes.

PREREQUISITES

The prerequisites list previous lessons that should have been covered before attempting the current one, as well as concepts it is assumed students have mastered.

TEACHING THE LESSON

This section makes suggestions as to how to go about teaching the lesson and often gives specific questions that can be asked. It discusses possible misconceptions and highlights concepts students traditionally have trouble understanding.

HOMEWORK AND ASSESSMENT

In Lesson 1.3, a suggestion is made as to how to assess the problems given for homework in the book. In general, two homework problems are provided for each lesson. These are optional and reinforce the ideas in the lesson.

ANSWERS (FOR BOTH THE STUDENT RECORDING SHEET AND HOMEWORK)

In-depth answers are provided for all questions asked.

EXTEND THE LEARNING

Extra problems are given for students who are looking for more of a challenge. Answers are provided.

DO I NEED TO DO THE LESSONS IN ORDER?

You should follow the order of the four sections, but you may decide not to do every lesson in every section. Once the class has achieved a comfort level with qualitative graphs (Section I), it can move on to quantitative graphs (Section II). The point at which this occurs may vary from class to class, so you may not need to go through every lesson in Section I if you feel your class has a firm grasp of the ideas. Likewise, you may move on to tables (Section III) and then equations (Section IV) without completing every lesson in the sections that precede them. Exercises in later sections that refer back to earlier lessons are clearly marked. On occasion, students seem to understand a concept but then lose sight of it later, when ensuing material is covered. If this should happen, you can easily go back to review a concept without disrupting the flow of the material.

The goal of this work is to give students a solid foundation in the graphical representation of functions which will help them throughout high school and college.