CHAPTER

2

The Tools of Classroom Talk

As teachers, we elicit responses from our students in various ways—with questions, commands, hints, jokes, and so on. When students become familiar with our inventory of phrases and expressions, they usually know what we expect of them. Although we rarely stop to think about our most common conversational prompts, they are among our most important instructional tools. From our work in Project Challenge, we have found it useful to think carefully about these tools: it matters what you say and how you say it.

In this chapter we present a number of examples of talk in action in mathematics lessons and describe the tools that teachers use to implement classroom talk. The tools include strategies—what we call "talk moves"—that support mathematical thinking, talk formats that provide different ways to organize students for conversation, and ideas for creating a classroom where respect and equal access to participation are valued norms. Narrative examples, or cases, illustrate how the basic tools of talk look in action in four different classrooms.

Five Productive Talk Moves

In this section we introduce five talk moves that we return to repeatedly throughout the book. Each move is a suggested action that we have found to be effective for making progress toward achieving our instructional goal of supporting mathematical thinking and learning. For each, we describe the move, give a brief classroom example to illustrate it, and then explain the teacher action required. Each move serves various purposes, and we describe some of those purposes in this chapter. However, you will discover other purposes for the moves in later chapters, and you may even discover some new purposes as you implement these moves in your own classroom. (It's important to note that these five moves are not the only ones that teachers can use to support productive mathematical talk. However, we focus here on these five as they provide a solid base on which to begin.)

Revoicing. **("So you're saying that it's an odd number?")**
When students talk about mathematics, it's often very difficult to understand what they say. Even if their reasoning is sound, it may not appear sound when they try to put their thoughts into words. Sometimes it's impossible to tell whether what they have said makes sense at all. And if you as the teacher have trouble understanding it, there's not much hope that the student's classmates will do any better. Yet given your goals of improving the mathematical thinking of all students, you cannot give up on an especially unclear student. If the only students whose contributions are taken seriously are those who are easy to understand, few students will ever improve. Deep thinking and powerful reasoning do not always correlate with clear verbal expression.

Therefore, teachers need a talk move that can help them deal with the inevitable lack of clarity of many student contributions. They need a tool that will allow them to interact with the student in a way that will continue to involve that student in clarifying his or her own reasoning. And they need a tool that will help other students continue to follow along in the face of the confusion. One such tool has been called "revoicing." In a revoicing move, the teacher essentially tries to repeat some or all of what the student has said, and then asks the student to respond and verify whether or not the teacher's revoicing is correct, as in the dialogue below.

Ms. Davies has given her third graders a series of numbers, and in a whole-group discussion has asked them to say whether the numbers are even or odd. They have established that if you can divide a number by two evenly, then it is an even number. Philipe has tackled the number 24. His contribution is less than completely clear.

1. Philipe: Well, if we could use three, then it could go into that, but three is odd. So then if it was . . . but . . . three is even. I mean odd. So if it's odd, then it's not even.
2. Ms. D: OK, let me see if I understand. So you're saying that twenty-four is an odd number?
3. Philipe: Yeah. Because three goes into it, because twenty-four divided by three is eight.

After hearing Philipe's confusing contribution, all Ms. Davies could grasp was that Philipe *might* be saying that 24 is odd. She hazards a guess in the form of a revoicing move: "So you're saying that twenty-four is an odd number?" By phrasing this guess as a question, she is essentially asking Philipe if her understanding is correct. By using this move, she gives him a chance to clarify. As it works out, he shows that he did intend to claim that 24 is an odd number, and he gives his reason. By opening this conversational space for Philipe to respond, Ms. Davies has learned that he has a basic misconception about even and odd numbers. She has gained a foothold in the discussion that she did not have after simply hearing Philipe's first contribution.

While revoicing is especially useful in situations such as that described with Philipe, it's also an effective move when you understand what a student has said but aren't sure that the other students in the class understand. Revoicing can make one student's idea available to others, give them time to hear it again, position a student's claim with respect to a previous student's claim in order to create the basis for an ongoing discussion, or focus on a change that has occurred in the discussion. Revoicing provides more "thinking space" and can help all students track what is going on mathematically.

Asking students to restate someone else's reasoning.
("Can you repeat what he just said in your own words?")
In the example above, the revoicing move was used by the teacher. However, the teacher can also extend the move to students, by asking one student to repeat or rephrase what another student has said, and then immediately following up with the first student. Ms. Davies used this move to continue the classroom conversation.

4. Ms. D: Can anyone repeat what Philipe just said in his or her own words? Miranda?
5. Miranda: Um, I think I can. I think he said that twenty-four is odd, because it can be divided by three.
6. Ms. D: Is that right, Philipe? Is that what you said?
7. Philipe: Yes.

This move has several potential benefits. First, it gives the rest of the class another rendition of the first student's contribution. It gives them more time to process Philipe's statement, and adds to the likelihood that they will follow the conversation and understand his point. It thereby supports the teacher's goal of giving all students full access to participation. This move is particularly valuable for students whose first language is not English. Second, this move provides evidence that the other students could and did hear what Philipe said. This is important: if students could not or did not hear what a speaker said, they cannot easily participate in further exchanges. Finally, it yet again clarifies the claim that Philipe is making and provides Philipe with evidence that his thinking is being taken seriously. Over time, as students come to realize that people are listening closely to what they say, they increasingly make efforts to make their contributions comprehensible.

Asking students to apply their own reasoning to someone else's reasoning. ("Do you agree or disagree and why?")

After a student has made a claim, and the teacher has made sure that students have heard it and have had time to process it, she can move on to elicit student reasoning about the claim. Ms. Davies employs this move to pursue the issue.

8. Ms. D: Miranda, do you agree or disagree with what Philipe said?
9. Miranda: Well, I sort of . . . like, I disagree?
10. Ms. D: Can you tell us why you disagree with what he said? What's your reasoning?
11. Miranda: Because I thought that we said yesterday that you could divide even numbers by two. And I think you can divide twenty-four by two. And it's twelve. So isn't that even?

By asking Miranda whether she agrees or disagrees with Philipe's claim and why, Ms. Davies is directing attention to Miranda's reasoning. Note that Ms. Davies has refrained from supporting one or the other position. At this point she is using talk moves to elicit respectful discussion of ideas. Later on, as students more fully understand the issues, Ms. Davies can make sure that they converge on the correct understanding of even numbers.

It's important to note that Ms. Davies does not simply ask whether Miranda agrees or disagrees but then follows up by asking her to explain why. As with the revoicing move, asking students to explain their reasoning is a key part of this move, and is critical in general to supporting students' mathematical learning. The point of this move is to cause students to make explicit their reasoning by applying their thinking to someone else's contribution. Therefore, she prompts Miranda to explain *why* she disagrees.

**Prompting students for further participation.
("Would someone like to add on?")**
At this point Ms. Davies increases participation in the discussion by asking for further commentary. First she uses the move of revoicing again as a way to clarify the two positions that have emerged, and to model how to talk respectfully to the originators of the two positions. Then she asks others to contribute, prompting them to either state agreement or disagreement, or to add other comments. This prompting for more input on previous statements will, over time, result in students showing more willingness to weigh in on what the group is considering.

12. Ms. D: So we have two different ideas here about the number twenty-four. Philipe, you're saying that twenty-four is odd because you can divide it by three?
13. Philipe: Uh-huh.
14. Ms. D: And Miranda, you're saying that it's even because you can divide it by two? Is that correct?
15. Miranda: Yes.
16. Ms. D: OK, so what about other people? Who would like to add to this discussion? Do you agree or disagree with Miranda's or Philipe's ideas? Tell us what you think, or add on other comments or insights.

Using wait time. ("Take your time . . . we'll wait . . .")
The final talk move we mention in this chapter is not actually speech at all, but silence! Many teachers are familiar with the important finding that after having asked a question, a teacher should wait at least ten seconds for students to think before calling on someone for an answer. Wait time also comes into play after a student has been called on. After a teacher has called on a particular student, that student should be given at least the same amount of time to organize his or her thoughts.

In the exchange above, after Ms. Davies has presented the class with a summary of Miranda's and Philipe's competing positions, and has asked for additional input, she waits . . . and waits . . . and waits. One or two students raise a hand immediately. Others look thoughtful, but don't volunteer. After five seconds, the students see that Ms. Davies is waiting for more responses. These students know that in their classroom it is not always the same superfast two or three students who will answer all the questions. They know that Ms. Davies will wait until a number of them think through her question.

After fifteen or twenty seconds, slowly, other hands go up. After forty-five seconds, Ms. Davies finally calls on Eduardo. He is hesitant, and actually sits silent after she calls on him even though his hand has been raised. So again, Ms. Davies waits. Ten seconds go by. Finally, the student responds:

17. Eduardo: Yes, I agree with Miranda's idea, because the only way you told us to find out if something is even is to divide by two. And if we divide twenty-four by three, we can also divide it by four. And we can divide it by six, too. So I think we should stick with two only.

By waiting patiently, Ms. Davies has made it possible for Eduardo, a second-language learner, to make an important contribution that she and other students can build on in the ensuing discussion. But this move is not easy for her. Although the research is clear on the value of wait time, it is actually quite difficult to adopt consistently. We all tend to feel uncomfortable with silence, not wanting to put a student on the spot. Yet few students can speedily put together an answer to a complicated question about their own reasoning. So if we do not use wait time consistently and patiently, students give up and fail to participate, knowing that they cannot "beat the clock." In later chapters we again address this and related moves designed to give students the time they need to think and reason mathematically.

Three Productive Talk Formats

Along with thinking about talk moves that guide students' learning, it's also useful to consider the talk formats available to teachers. Talk formats are different ways that teachers configure classroom interaction for instruction. For example, Ms. Davies took advantage of all five talk moves and used the talk format of whole-class discussion, having her entire class participate together in mathematical thinking and reasoning. Every classroom teacher makes use of a variety of talk formats, and these formats are among the major tools that teachers use to accomplish their goals for student learning. Each format carries with it certain opportunities and certain limitations.

Each format has its own "rules for talk." Some of these rules are rarely discussed, but students know them, nevertheless. For example, in the traditional and familiar talk format that we might label *direct instruction* or, in the higher grades, *lecturing*, the rules go something like this: the teacher has the right to talk, and students must not talk unless the teacher calls out their name. *Quizzing* is another commonly used talk format in which the teacher asks questions for which he or she knows the answers and expects the students to know the answers as well. The rules are as follows: the teacher calls on a student and evaluates the answer given as to its correctness. "Jamie, how much is three times eight?" "Twenty-four?" "Good." In research on classroom talk this is called the IRE format, for *initiation* (by the teacher), *response* (by the student), and *evaluation* (by the teacher). Other talk formats include *sharing time, group recitation,* and *student presentations.*

While there are many academic purposes that may be served by using these formats for mathematics instruction, in this book we focus on three talk formats—whole-class discussion, small-group discussion, and partner talk. We have found these talk formats to be particularly helpful in maximizing opportunities for mathematical learning by all students.

Whole-class discussion.

The talk format that appears most prominently in this book is whole-class discussion. In this talk format, the teacher is in charge of the class, just as in direct instruction. However, in whole-class discussion, the teacher is not primarily engaged in delivering information or quizzing. Rather, he or she is attempting to get students to share their thinking, explain the steps in their reasoning, and build on one another's contributions. These whole-class discussions give students the chance to engage in sustained reasoning. The teacher facilitates and guides quite actively, but does not focus on providing answers directly. Instead, the focus is on the students' thinking.

It takes students a great deal of practice to become solid and confident mathematical thinkers, and this talk format provides a space for that practice. In whole-class discussion, the teacher often refrains from providing the correct answer. He or she does not reject incorrect reasoning, but instead attempts to get students to explore the steps in their reasoning, with the aim that they will gain practice in discovering where their thinking falls short. Invariably, these discussions reveal many examples of faulty reasoning, mistakes in computation, and misunderstandings. These flaws, however, are the raw material with which teachers can work to guide students' mathematical learning. And, in the process, students become more confident in their ability to stick with making sense of concepts, skills, and problems. They gradually lose some of the anxiety and avoidance behaviors that many students display when confronted with complex mathematical ideas.

The purpose of whole-class discussion is to provide students with practice in mathematical reasoning that will further their mathematical learning. To accomplish this, the focus is on the students' ideas, not on the correctness of their answers. This does not mean that we are advising teachers to deemphasize correct answers and mathematical truth. In our view, the ultimate goal is for students to achieve mathematical power through precision, accuracy, insight, and reliable reasoning. However, we have found that it's important for students to have opportunities to practice their reasoning in discussions without an immediate focus on correct answers.

How can students' learning of mathematics be supported if teachers don't let them know when their thinking is misguided or an answer is incorrect? Aren't there times when it's better to tell students when their answer or idea is wrong? To answer these questions, it's important to think about what learning of mathematics involves. More specifically, when confronted with any new mathematical concept or skill, it's important to consider where the source of the knowledge is for the student.

Sometimes the source of mathematical knowledge lies outside a student and the only way that a student can have access to the knowledge is from an external source, such as a book, a television program, the teacher, or another student. For example, the mathematical symbols we use to represent ideas are socially agreed upon conventions, and the source of learning these symbols lies outside the student. There is no way for a student to "discover" the meaning of a plus sign—we show it to students and tell them what it means. The same is true for the operation signs for subtraction, multiplication, and division, for the relational symbols for equal, greater than, less than, even for the way we write the numerals. These symbols have no meanings that are inherent to them but rather are mathematical conventions that we all agree to use for ease of representing and communicating mathematical ideas. The same is true for the terminology we apply to mathematical ideas—*triangle*, *prime number, even, fraction*, and so on. When mathematical knowledge is linked to social conventions, direct instruction is appropriate for furthering students' learning.

When mathematical concepts and skills are not linked to social conventions, but rather, have their own internal logic, the source of knowledge is not external to the student. Instead, students learn by processing information, applying reasoning, hearing ideas from others, and connecting new thinking to what they already know, all for the goal of making sense for themselves of new concepts and skills. The source of the knowledge, of creating new understanding, lies within the student, and making sense is the key. We can tell a student, for example, that the order of the numbers in a multiplication problem doesn't alter the answer, that 2×5, for example, produces the same product as does 5×2. But this is an idea that students can figure out for themselves, from experimenting with problems, thinking about what happens to the products when factors are reversed, and then talking about their ideas, as the students in Mrs. Schuster's class did in the example presented in Chapter 1.

Dictating to students in direct instruction is not appropriate for teaching ideas for which the source of the knowledge is inside the student. In order for children to learn, understand , and remember, they need experiences interacting with the idea, thinking about it in relation to what they already know, uncovering its logic, and then applying their thinking to this new idea. Using talk in a whole-class discussion provides students the opportunity to make sense of new ideas. Such discussions may also reveal students' confusion, partial understandings, and misconceptions, important information for teachers to have when planning instruction. Explaining their reasoning is important for all students as it helps them to cement and even extend their thinking. Over the long run, we have seen that being asked "Why do you think that?" has profound effects on students' mathematical thinking and on their "habits of mind" in general.

Small-group discussion.

In this book we make a distinction between whole-class discussion and small-group discussion. In the small-group-discussion format, the teacher typically gives students a question to discuss among themselves, in groups of three to six. While the rules for whole-class talk formats are generally familiar to students, students need help becoming familiar and comfortable with the rules for small-group discussion, since multiple conversations occur. In this format, the teacher circulates as groups discuss and doesn't control the discussions, but observes and interjects. The teacher necessarily plays a diminished role and therefore cannot ensure that the talk will be productive. Students can spend time on off-task talk, and there is no guarantee that students will treat one another in an equitable manner. This format has many important functions in mathematics class. However, it figures less prominently in this book. Our focus here is on how teachers can actively create conditions for talk that will be reliably productive in terms of mathematical thinking and reasoning.

Partner talk.

Small-group discussion is distinct from what we call *partner talk*. In this talk format, the teacher asks a question and then gives students a short time, perhaps a minute or two at the most, to put their thoughts into words with their nearest neighbor. This format has several benefits: students who are keeping up with the lesson but are hesitant about voicing their thoughts will have a chance to practice their contribution with just one conversational partner. Students who have not understood completely can bring up their questions with the partner, and perhaps formulate a way to ask them to the class. For many students, particularly those who are learning English as a second language, this one- or two-minute aside is invaluable. They can emerge from the partner talk ready to participate in the whole-group discussion. When wait time doesn't increase the number of students willing to talk, changing the talk format to partner talk can help. A teacher can initiate partner talk by saying to the class, "Turn and talk about this with the person next to you." This results in a much noisier class, but the noise has benefits as more students think out loud. After a few moments, the teacher can stop the partner talk and return to the format of whole-class discussion.

Ground Rules for Respectful Talk and Equitable Participation

First and foremost, before any talk moves can be implemented, the teacher must establish ground rules for respectful and courteous talk. You will not be able to use successfully the moves described previously unless you have established a classroom culture in which students listen to one another with respect. If students are afraid that their ideas will be ridiculed, they will not talk freely, no matter what inducements you offer. They must feel that their classroom is a safe place to express their thoughts. Therefore, your first steps in creating the conditions for productive classroom talk must involve setting up some clear ground rules for interaction. It is very important to put this step first. Even one hostile or disrespectful interchange can put a serious damper on students' willingness to talk openly about their ideas and thoughts. It is imperative that you consistently maintain the ground rules for respectful and courteous talk, and that your students know that there will be no exceptions.

The ground rules must center on each student's obligation to treat one another with respect. No name-calling or derogatory noises or remarks are ever allowed. "I was just joking" cannot be an acceptable defense for a disrespectful remark, and students must know that you will hold them to a high standard. There must be clear consequences for violation of these rules. You may need to remind students of these rules every day until they become a routine part of your classroom culture. We recommend creating a poster or wall chart and prominently displaying it so that you can make reference to it when necessary.

As you establish the conditions for respectful and courteous talk, you will also need to set the conditions for full participation: all students must have the opportunity to engage in productive talk about mathematics. This means that you must make sure of three things: (1) that every student is listening to what others say, (2) that every student can hear what others say, and (3) that every student may participate by speaking out at some point.

As part of your ground rules for respectful and courteous talk, you no doubt will have put in place a rule that obligates students to listen attentively as others talk. This is respectful behavior, but just as important, it is pragmatic behavior. It enables students to participate in the ongoing talk. If they do not know what was just said, they cannot possibly build on it. While establishing these ground rules may sound difficult, many teachers have had success within the first few weeks or months of teaching this way. In Chapter 9 we introduce specific suggestions to help you put these norms in place.

Four Cases: What Does Productive Talk Look Like?

The following narrative examples, or cases, show you how talk moves look in action in the classroom. These cases, or examples, are composites based on actual classes we have taught or observed. The examples in this chapter are from grades one, three, five, and six. Each case begins with a brief description of the mathematical ideas or problems that are central to the lesson, along with a description of what the class has done so far. We then present the interaction in the form of a script, tracking the teacher's and students' contributions to the conversation.

Case 1. (**"I disagree with Juana's solution because four won't work."**)
In Ms. Day's third-grade class, the students had been finding solutions for pairs of equations that are number sentences in which unknowns are indicated by squares and triangles. For each pair of equations, students must figure out one value for the square and one value for the triangle that makes both sentences true. For example, in this case, students had been considering the two number sentences shown below and have been asked to figure out what values of the square and triangle make both equations true.

$$10 - \square = \triangle$$
$$\square + \square + \triangle = 13$$

In order for both sentences to be true, the square must have a value of 3 and the triangle must have a value of 7.

A problem like this is useful for a number of reasons. First, it gives children practice with arithmetic as they try out different numbers to make both sentences true. Second, it provides children early exposure to two ideas that are central to algebraic thinking—finding a value for an unknown, and finding a solution that simultaneously solves two different equations. Third, the problem is accessible to almost all students because it can be solved using a guess-and-check strategy. Third graders have no way to manipulate these equations algebraically, so they have to start with the strategy of guessing two numbers that might work, and plugging them in to check whether they do work. Finally, because there are two number sentences for which the numbers must work, students must perform the logical operation of checking whether the numbers they choose work for both sentences.

Ms. Day knows that some of the students understand that they are finding a solution that works for both equations, while others do not. Although she has explained to them that the value they choose for the square and triangle must remain the same for both sentences, many of them do not seem to comprehend her explanation fully. She decides to use a session of classroom talk to help them.

First, Ms. Day gives the students five minutes to work on the problem individually. As they work, she circulates through the room looking over their shoulders. She notices that Juana has not grasped the connection between the two number sentences, while Jaleesa has already solved the problem. David seems to be wavering between treating each number sentence as an isolated problem and connecting the two as one system to be solved. Ms. Day refrains from giving students any assistance during this time. After the five minutes are up, she starts the whole-class discussion session with a statement:

1. Ms. D: I'd like someone to explain the solution you got, and I'll write it on the board. Then we can see who agrees, who disagrees, who has the same answer, or who has a different answer. Juana, what was your solution?
2. Juana: Umm, I think the square can be six, and then the triangle is four.
3. Ms. D: [Writes on the board:] $\square = 6$, $\triangle = 4$. OK, so tell us what your reasoning is.
4. Juana: Well, if you put them in the sentence, then ten minus six is four.
5. Ms. D: OK, so let's hear what people think. Did other people get the same answer, a different answer, or what do you think

about what Juana said? Jaleesa, your hand is up. Did you agree or disagree with Juana's solution? And tell us why.

6. Jaleesa: Well, I sort of disagree, because four doesn't work.

7. Ms. D: Hmm, so you think four doesn't work. But look, in this sentence here, it does work. Juana says the square is equal to six. Ten minus the square, which is six, equals the triangle, which is four. That works, right? So what doesn't work? I'm confused. Can someone else explain what Jaleesa is saying? How about you, David?

8. David: I think I know what Jaleesa is saying. She's saying that four doesn't work in the *second* sentence—square plus square plus triangle equals thirteen.

9. Ms. D: So, Jaleesa, is that what you were saying?

10. Jaleesa: Yes, because in the second sentence if you use four for the triangle then the two squares will have to equal up to nine and that can't work. Because in the first one they're six.

11. Ms. D: Wait, I'm not sure I'm following that. I'm a little confused here. You're going kind of fast. David, do you want to try to say what Jaleesa said in your own words?

12. David: OK, I think she means that the triangle number has to be the same number for the first sentence *and* the second sentence. And the square number has to be the same for both sentences too. So if you put four in for the triangle on the first sentence, then you have to put four in for the triangle on the second sentence. And that won't work.

13. Ms. D: That won't work? Is that what you meant, Jaleesa?

14. Jaleesa: Yes, because . . . can I show you on the board?

15. Ms. D: Sure. Come on up.

Notice that Ms. Day has used the talk move of revoicing in several different ways here. In Line 7 she asks David to revoice Jaleesa's position, and then she herself completes the revoicing move by asking Jaleesa, in Line 9, if he is correct in his summary of her position. In Line 11, she models confusion, and asks him to once again "put it into his own words." Then she again checks with Jaleesa. By the time Jaleesa comes to the board, her own position has become clearer to her and she is eager to share it.

16. Jaleesa: OK. [She writes on the board:]

$$\boxed{10} - 6 = 4$$

OK, this one works. But now we have to put the same numbers in the second sentence and it has to work too. [She writes:]

$\boxed{6} + \boxed{6} + \triangle\!\!\!\!\!4\; = 13$

But this one doesn't work. [She sits down.]

17. Ms. D: OK. Hmm. So Juana, what do you think about that? Do you agree or disagree with what Jaleesa said? And tell us why.

18. Juana: Well, I guess I didn't think that we had to put the same numbers in both sentences. I thought we could use different ones.

19. Ms. D: Yes, this kind of problem is harder than just filling in one sentence, isn't it? We have to fill in two at the same time! We have to make sure that both sentences are true. Let's do another one. This time you can spend three minutes working with the person next to you. [Ms. Day writes two different equations on the board:]

$$\square - \triangle = 15$$
$$\square + \triangle + \triangle = 21$$

Recognizing and understanding the teacher's moves

A great deal is going on in this series of exchanges. Let's focus on the teacher's purposes and her use of talk. While many teachers already use the instructional strategy of asking students to share their solution to a problem, our focus in this case is on Ms. Day's use of talk to bring the students to a common ground of understanding. She knew that not all students had understood the constraints of the problem, even though she had done her best to explain it. Her immediate purpose was to clarify one aspect of the math problem: the problem solver has to consider *both* sentences simultaneously in order to find a valid solution. She had observed that some students, like Jaleesa, understood this. Some students, like David, were wavering. And still others, like Juana, did not understand this constraint.

This situation, where different levels of understanding can be found side by side in the same class, is extremely common and is always a challenge. It is one reason that teachers sometimes shy away from carrying out a whole-class discussion. How can we have a discussion involving all students if only some of them understand the problem? But Ms. Day used the students' contributions in a systematic fashion to clarify a misconception that she suspected many students held, not just Juana.

So why did Ms. Day say what she did? Why did she call on the students she called on? Ms. Day first brought the misconception to the forefront by asking Juana to present her solution. She did not correct Juana, but asked the students in an open, neutral way whether anyone agreed or disagreed with the solution. She knew that Jaleesa would disagree, and would present some form of argument against Juana's claim. So she called on Jaleesa. Jaleesa did disagree, but did not provide a full explanation of why she disagreed with Juana's solution. Right after Jaleesa gave this partial explanation, Ms. Day expressed confusion.

Why does Ms. Day say in Line 11 that she is confused? Was she really confused? No, but she knew that other students were. Jaleesa's explanation was quick and Ms. Day knew that it would be difficult for some of the other students to follow. An explanation that cannot be understood is not worth much to the other students. In instances like these, the teacher must try to expand the time and space available for the class to understand what one student already knows. So here Ms. Day makes a skillful move: she brings a third student into the conversation, David, who is not entirely clear about the constraints of the problem. By asking him to revoice Jaleesa's explanation, she is hoping to draw him (and others) into Jaleesa's thinking and lead him to a fuller acceptance of the two-sentence constraint on the problem. Finally, after David helps construct a fuller explanation, Ms. Day returns to Juana, and allows her to revisit her original analysis. Then, by moving on to another, similar example, and allowing students to talk to one another as they solve it, Ms. Day is using talk in yet another way to solidify students' understanding.

Ms. Day's use of whole-class discussion

It takes students a great deal of practice to become solid and confident mathematical thinkers, and the talk format of whole-class discussion provides a space for that practice. Accordingly, within a class discussion, the teacher often refrains from providing the correct answer. The teacher does not reject incorrect reasoning, but instead attempts to get students to explore the steps in that reasoning, with the aim that they will gain practice in discovering where their reasoning falls short. In this case, Ms. Day did not correct Juana's misconception directly, but instead asked other students to react to what Juana had said. As we said earlier, whole-class discussions will reveal many examples of faulty reasoning, mistakes in computation, and misunderstandings. Ms. Day decided to use these as her raw material to help her students develop their problem-solving abilities.

When Ms. Day refrains from correcting the students in this part of the discussion, she is not losing sight of her ultimate goal: that her students achieve mathematical power through precision, accuracy, insight, and reliable reasoning. She knows, however, that her students need to practice their reasoning in complex problem solving without an immediate focus on the answer. After this discussion, Ms. Day will summarize and review the correct solutions, along with the generalizations that the group has made based on their discussion.

Case 2. ("This is a triangle because it just looks right.")

Ms. Sigler's first-grade students are learning about geometric figures. The students have been given a set of polygons to sort into two groups: triangles and other shapes. Within the set there are many different sizes and kinds of triangles, including scalene, equilateral, isosceles, and right triangles. The triangles are cut out of colored paper and are either red, green, or blue. Other shapes are also in the set—squares, rectangles, hexagons, and parallelograms. Students are working in groups of four. Ms. Sigler notices that one group of students has separated the equilateral triangles from all of the other triangles and grouped them alone. They have placed most of the other triangles in the "other" category.

1. Ms. S: I see that you have separated the shapes. What is this group of shapes called? [Points to the set of equilateral triangles.]
2. All: Triangles.
3. Ms. S: I'd like each of you to explain your thinking to me. I'd like each of you to explain to me why all of the shapes in this group are triangles [points to the equilateral triangles] and all of these are not triangles [points to some triangles in the other group].
4. Ollie: This [points to one equilateral triangle] just looks like a triangle.
5. Ms. S: In what way?
6. Ollie: It's short and fat and just looks right.
7. Yoon So: Yeah. This one [points to a scalene triangle] is too skinny and pointy. It isn't a triangle.
8. Ms. S: What do you think, Paul?
9. Paul: Well, you know these are maybe triangles [indicates the group of equilateral triangles] because you know they just look like triangles.
10. Ms. S: What does a triangle look like?
11. Paul: Hummm. [Paul looks around the room and then points to a poster behind the door of geometric shapes. The shapes on

the poster are all regular polygons and the triangle on it is equilateral.] Like that one. See, triangles look like this [puts thumbs and forefingers together to make the shape of an equilateral triangle]. They're flat on the bottom.

12. Ms. S: Ollie, what did Paul just say?

13. Ollie: These are a triangle [uses his hands to make the same shape Paul made].

14. Ms. S: What else did he say?

15. Ollie: They have to sit on their bottoms—there aren't pointy parts on the bottom.

16. Ms. S: OK. Look at this. [She takes one of the large equilateral triangles from the triangle pile and turns it so a vertex is pointing downward.] Is this a triangle?

17. Paul: [Turns the triangle so it is sitting on a side.] Now it is.

18. Ms. S: Sim, you have been very quiet. What do you think? Is this a triangle? [Turns the equilateral triangle around so the point is again facing down.]

19. Sim: I don't think so. I don't know.

20. Ms. S: Why don't you think it's a triangle?

21. Sim: I don't know. [Long pause.] Maybe it is. Because I can turn it and it looks like a triangle.

Ms. Sigler has been asking each student in turn to justify their categorization of the equilateral triangles as the only "real" triangles. Ms. Sigler has seen that the children in this small group have a restricted idea of what a triangle is. This is quite typical for young children: their definitions for geometric shapes are based on their visual memory of their previous experiences with that shape. Equilateral triangles are typically used to illustrate the concept of "triangle." One teacher we know calls this the "Sesame Street" level of understanding geometric shapes.

Some readers might find this small-group interchange to be somewhat slow and laborious. Is the teacher really moving the children toward changing their understanding? Yes, but first she must assess what they believe, find out how they think about the question. And to do this, she is getting them to *externalize* their ideas by asking them each to speak in turn. Notice that with each new contribution, their low level of understanding gets a bit more obvious. Even though the teacher pushes them to justify their categorization of the triangles, they offer simplistic reasons like "it looks right." It doesn't seem that the talk is moving the group very far into new understandings, but at this stage Ms. Sigler does have a pretty clear picture of the level of this group's understanding. She sees that this classification task has not been an effective instructional activity up to this point.

The next day, Ms. Sigler brings out a large box of cardboard strips of different lengths, with holes at the ends, which can be fastened together by the use of brads. From the previous day's discussion she knows that the students are simply identifying the names of geometric shapes with a typical image. They are not considering the defining properties of a triangle, such as the fact that a triangle is any closed figure with three straight sides. She gives each child three strips from the box, and asks them to use the strips and the fasteners to create a triangle. In grouping the strips, she has made sure that all varieties of triangles—equilateral, isosceles, and scalene—will be produced by the students in the class.

After each child has made a triangle, Ms. Sigler tapes them all to the front board (see Figure 2–1). She then holds a large group discussion about the triangles.

1. Ms. S: Let's look at these. Does anybody notice anything about the shapes? Cecile?
2. Cecile: All of them are made from three strips.
3. Ms. S: Does anybody see a triangle that was made from more than three strips?
4. Students: No, no.
   They're the same.
   They all have three.
5. Ms. S: I notice that all of these triangles are not identical; they don't all look the same. How are they different? Pooja?
6. Pooja: Some of them are big and some are small.
7. Ms. S: What makes a triangle big or small? See these two triangles? [Mrs. Sigler takes one large triangle and one small triangle and places them next to each other on the board.] What did someone do to make them different?
8. Pooja: You use long strips to make the big one and short strips to make the small one.
9. Paul: Mine is the big one and I used the really long strips.
10. Ms. S: How many long strips did you use, Paul?
11. Paul: Three.
12. Ms. S: Who else made a big triangle? [Points to a couple of large

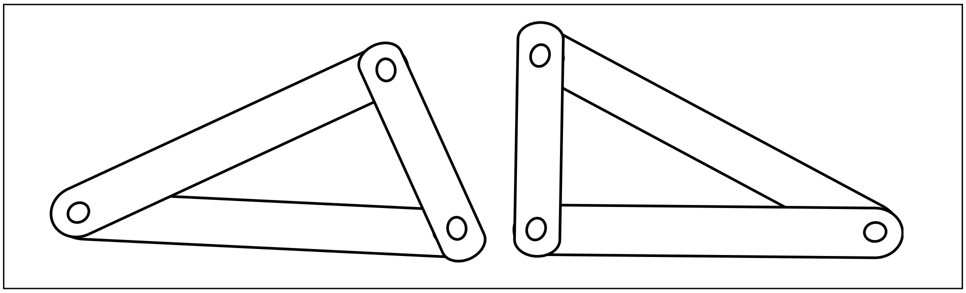

FIGURE 2–1 **Triangles constructed using cardboard strips.**

triangles on the board.] How many strips did you use?

13. Cristobal: I made that one over there. [Points to an isosceles triangle.] I used three strips.

14. Ms. S: Who made a small triangle?

15. Ben: I did! I used three strips, too!

16. Ali: Me, too. My triangle took three strips.

17. Ms. S: OK, so now let's use a word that people use to talk about some geometric shapes. We can use the word *side*. This triangle has three sides—one, two, three. [Ms. S points to each side as she counts.] How many sides does Ali's triangle have? [Here Ms. S points to Ali's triangle, a long and thin scalene triangle made with two longer strips and one short one.]

18. Ali: My triangle has three sides.

19. Marsha: But Ms. Sigler, I don't know if that's a triangle. It doesn't look like the triangles from yesterday.

20. Ms. S: That's an important question. Can somebody repeat for us what Marsha just said? Ali, can you repeat what Marsha just said?

21. Ali: She said that my triangle isn't really a triangle because it doesn't look like the ones we put in the triangle group yesterday.

22. Ms. S: Marsha, is that what you said?

23. Marsha: Yes, but I didn't mean that Ali's triangle wasn't a triangle. I just meant that it didn't match the ones from yesterday in our group.

24. Ms. S: OK, so Marsha has asked a really important question. Is this shape a triangle? And what about those triangles from yesterday? Remember yesterday, when we were sorting triangles? Some of you thought that a shape like this [points to

> Ali's triangle] didn't belong in the same group with triangles that looked like this [points to an equilateral triangle on the board]. So we have a really important question here. Just what *is* a triangle?

Ms. Sigler has brought the discussion to a very important point: the children's attention is now focused on whether or not all of the shapes they have constructed are triangles. She now has several choices. She could tell them that Ali's shape is a triangle and ask them to figure out why. She could ask them what they think and see where the discussion leads. She could provide some direct instruction about the properties of triangles, starting with the fact that they have three sides and are closed figures and then asking the students to check whether all of the figures on the board meet those two conditions.

Ms. Sigler has succeeded in using classroom talk to bring these first graders to a point where they are ready to engage with the idea that triangles that are not equilateral triangles are also properly called triangles. What talk moves did Ms. Sigler use with these first graders? Notice that she asked many students to reiterate the idea that three strips are needed to make a triangle, even though everyone had agreed that all the shapes on the board had used three strips. Students need time to generalize important ideas; in this case they had to consider the fact that all of the different-looking shapes have three sides. Although this idea is less complex than those we describe in some other examples in this book, it's an important idea for these first graders. They deserve the time that it will take to make sure that everyone has made the same generalization.

The slow and gentle nature of the talk in this first-grade classroom is also reflected in other ways. Ms. Sigler lets these students spend time in their small groups, working with an activity while she circulates among the groups. Then she moves the activity to a whole-class format, making sure that all students have had a chance to think about and talk about the material that will be discussed in the large group.

Notice that although Ms. Sigler makes sure to give students time to consider the problem and to listen to one another, she does not shrink from difficult material. For example, Ms. Sigler does not hesitate to introduce correct mathematical terminology. She uses the term *geometric shapes* and she directs the students to use the word *side* when talking about their triangles. She could have spent even more time with this, particularly if she had had many English-language learners in the class.

Finally, it is important to note that even with these young children, Ms. Sigler allowed the students to consider her questions for quite some time without providing them with answers. She could have started the first day by simply telling the students that they were wrong in their categorizations, that all three-sided closed figures are triangles. In some situations one might want to proceed in this way, and we would not rule it out as an option. But Ms. Sigler chose the more indirect route, in the belief that letting students follow the ideas at their own pace and in their own way would more likely result in their being ready to face the question that Marsha brought up. Her ultimate aim is to use classroom talk to bring students to understand the criteria for what makes a shape a triangle, thereby moving them beyond their *Sesame Street* understanding of this geometric shape.

Case 3, Episode 1. (**"So you drew a picture of the tarts?"**)

Ms. Stangle's fifth-grade class is focusing on a problem that is designed to give them experience with fractions:

> *Ms. Stangle wants to make peach tarts for her friends. She needs two-thirds of a peach for each tart and she has 10 peaches. What is the greatest number of tarts that she can make with 10 peaches?*

Ms. Stangle has the students work on the problem on their own for ten minutes. At the end of the ten minutes, she doesn't know what types of solutions students have come up with, because she had an unexpected visitor to the classroom and did not get a chance to circulate and look at their work. However, she expects that some students had difficulty setting up the problem, others had difficulty representing the facts accurately, and still others struggled with computation.

1. Ms. S: Who wants to present their solution? How about some hands here? OK, Marco.
2. Marco: Well, first I looked at them all and then I made lines on each one and then I counted.
3. Ms. S: Marco, I'm not really sure I follow you. It sounds like you drew a picture of the tarts and the peaches, is that right?
4. Marco: Well, yeah.
5. Ms. S: We can't really follow what you say about your solution unless we can see the picture in our minds. We have to be able to listen to you and really understand what you're describing. Would you like to tell us again about the details of what you drew?

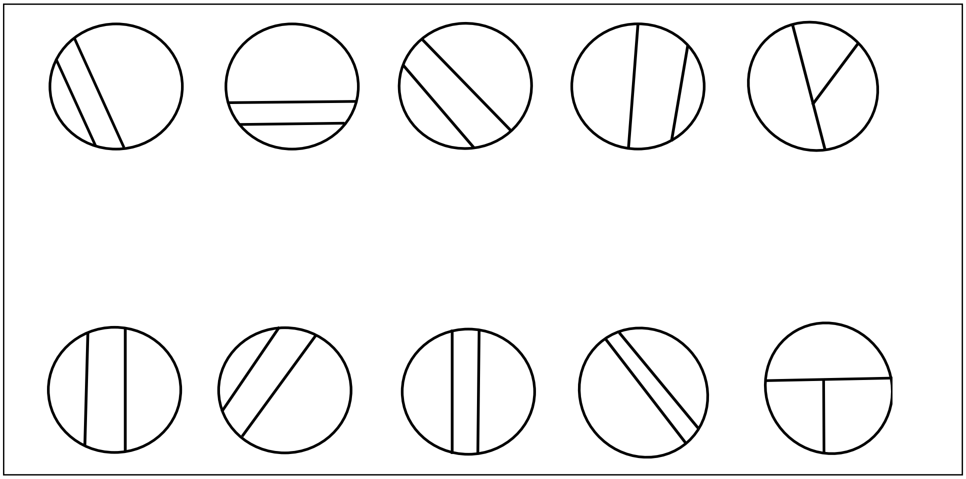

FIGURE 2–2 **Ms. Stangle's representation of Marco's solution.**

6. Marco: Um, OK. I drew the ten peaches and then I cut each one into three parts. Then I counted all the parts. So it was thirty parts. And the problem says that each tart needs two parts of a peach.

7. Ms. S: Hold on, I'm getting lost again. I thought the problem said . . . well, wait. So you're saying that I can take any two parts of a peach and that will be enough for a tart? Is that what you're saying?

8. Marco: Two of the three parts.

9. Ms. S: OK, so let me see if I can draw what you're describing. Here are my ten peaches [draws ten circles] and here I'm dividing each one into three parts [draws unequal partitions of each peach; see Figure 2–2]. Did anyone else have a picture like this?

10. Students: Nooo!
Wait!
Not like that!

11. Ms. S: No? How come [innocently looking surprised]? What's wrong with this? I've cut each peach into three parts.

12. Ginny: No, it has to be equal or it won't be thirds.

13. Ms. S: Oh, so the problem says it has to be thirds? Is that right?

14. Ginny: Yes.

15. Ms. S: OK, so Marco do you want to add to your statement of your solution?

16. Marco: OK. I drew ten peaches and I drew lines that cut them

into *equal* thirds! And then I drew ten tarts and I matched them up.

Recognizing and understanding the teacher's moves

You might wonder why Ms. Stangle didn't just ask Marco to draw a picture on the board. After all, it's good mathematical practice to have students present their graphic representations to the class. What is Ms. Stangle trying to do here? Why is she subjecting the students to this ordeal of clarification? Why is she being so cautious in making sure that their meaning is absolutely clear? She could not be as obtuse as all that—surely she knew that Marco knew he was counting thirds, not just random parts of a peach.

In our work we have found that one of the most effective ways to increase students' attention to precise language is to engage in just such episodes as the fragment previously presented. When the teacher sets a high criterion for clarity, when she herself "fails" to understand what the students are saying until they are utterly clear, she is modeling for them an attention to detail in language that will serve them in a variety of ways. Experiences like these will eventually lead students to use language more precisely, because it raises their level of awareness of the audience. They will also begin to similarly scrutinize the language of others. We have seen this kind of awareness lead to clear gains in reading comprehension, both in mathematics and in English language arts.

In this excerpt Ms. Stangle skillfully uses the technique of revoicing. She repeats what the student has said, clarifying, adding, or making more obvious some problematic feature of it. She then asks the student whether that is the correct interpretation. In the first few lines, she uses this conversational move in order to get Marco to clarify his first statement from Line 2. Like many students, Marco is not particularly skilled at anticipating what his audience will need. An outsider to the classroom would not understand him when he says, "Well, first I looked at them all and then I made lines on each one and then I counted." And although the other students have been working on the same problem, there is no guarantee that their solutions look the same. Therefore, Marco's description has to be clear enough for others to envision exactly what he has done, and to connect it to what they have done.

Ms. Stangle could have asked Marco to draw his picture on the board, and in fact she often has a student do just that. But in this excerpt, she skillfully uses classroom discourse to get Marco to reach a higher level of verbal precision. In Line 9, she takes his words literally and draws a picture that matches what he has said (although it does not match his intentions). As other students see the discrepancy, they understand the problem and want to correct it.

Case 3, Episode 2. ("Thirds of tarts or thirds of peaches?")

17. Ms. S: Can somebody repeat what Marco did for his solution so far? Cheryl?

18. Cheryl: I think he said he drew a picture of the ten tarts, and then he, like, drew a picture of the ten peaches. Then he cut the tarts into three pieces each. And he drew, like, lines to the peaches.

19. Ms. S: OK, so does everyone agree with Cheryl's rendition of what Marco said? Was that what Marco said? Does anyone want to weigh in here? Ginny?

20. Ginny: Well, it was almost the same, but Cheryl said that Marco drew ten tarts and cut them into three pieces each. And I think he said that he drew ten *peaches* and cut them into three pieces, three thirds, each.

21. Ms. S: So Marco, would you clarify? Which one did you do? Did you cut the tarts into thirds in your picture or did you cut the peaches into thirds?

22. Marco: Umm . . . I'm not sure now.

23. Ms. S: OK, who can help out? Did anyone else have a picture where they divided either the tarts or the peaches into thirds? Tyavanna?

24. Tyavanna: I drew ten peaches? And . . . um . . . like . . . um, I think that's what you have to do because you have to show the thirds of the peaches? 'Cause that's what you have to figure out to make the tarts. Not the other way around.

25. Ms. S: OK, can somebody repeat what Tyavanna just said? Kenny, can you repeat it in your own words?

26. Kenny: Well, I think she's saying that we have to draw pictures of the ten peaches so we can draw the thirds of each peach. She's saying we don't have to draw thirds of each tart. But I didn't draw the peaches at all.

27. Ms. S: OK, so you did it a different way? Will you tell us how you started out?

28. Kenny: I wrote the fraction *three-thirds* and I wrote it once for each peach. Then I added up all the three-thirds and I got thirty-thirds. So then I knew how many thirds I had to work with. Then I knew that each tart had to have two thirds, so I just divided two into thirty and I got fifteen. So I knew there could be fifteen tarts.

Recognizing and understanding the teacher's moves

Notice how at the beginning of episode two, in Line 17, Ms. Stangle asks Cheryl to repeat what Marco said. This move quickly puts students on notice that they must listen to their classmates' contributions and cannot simply sit and wait for their turn. They must actively engage in trying to understand what others are saying. In fact, Cheryl incorrectly repeats what Marco said, perhaps because she does not understand or perhaps because she has made a simple speech error. We do not know which is the reason for her error, and neither does Ms. Stangle. Nevertheless, Ms. Stangle brings other students into the process of clarifying and correcting. As part of this process, she asks Tyavanna to explain her understanding, and Tyavanna is able to shed some light on the situation.

Why does Ms. Stangle choose to ask Kenny to repeat what Tyavanna has said (Line 25)? Is it because she thinks he isn't listening and she wants to call him back? Or is it because she wants to make sure that all students have heard and understood what Tyavanna has just said? Ms. Stangle could have both intentions: her skillful use of this request for students to repeat could serve both functions simultaneously. But some readers may ask whether this set of moves—the questioning about details and the requests for repetition—isn't awfully time-consuming, and perhaps even annoying. How could one run every class this way? Who has the time? Doesn't this focus on precision drive people crazy?

Ms. Stangle does use these techniques with regularity, but there are many times during lessons when she does not require students to achieve the same degree of precision as in episode one. For example, when students encounter a new idea, and are trying to come to grips with it, their language typically becomes imprecise and even incoherent. A moment's reflection reveals that this is true of adults as well—when we are dealing with new concepts and unfamiliar ideas, any of us may sound quite inarticulate! As the cognitive demands on us increase, our ability to talk precisely may deteriorate precipitously. Knowing this, teachers like Ms. Stangle strike a delicate balance. She does not require absolute precision of students as they are working through new ideas. When they are engaged in the most demanding kinds of thinking, she is less stringent in her attention to clarity and precision. On the other hand, when students are more comfortable with the mathematical ideas, she can afford to emphasize the precision of their language. Her students encounter these demands for precision often, and thereby come to understand and accept the need for precision. They develop an appreciation for the effort we all must make to communicate clearly, in mathematics and elsewhere. And although at first students do not like to have to repeat what other students have just said, over time they become remarkably attentive and skilled at keeping track of where the conversation is going. It is in just such settings that teachers are able to eventually achieve real advances in mathematical understanding through talk.

Case 4. ("Discuss this with the person next to you.")
When we divide a whole number by a fraction, such as 2 divided by $\frac{1}{4}$, the quotient, 8, is larger than the dividend! For most students, this is a confusing new result that does not connect well with their previous experience of division. In lower grades, division problems typically yield answers that are *less* than the dividend. Also, this problem does not lend itself to being interpreted in the way students more often interpret division situations, as "sharing." How can you share two pies among one-fourth of a person? And what does an answer of 8 mean in terms of the most common situations associated with division, like sharing? In the face of this departure from the students' favorite meaning of division, many teachers resort to simple computational rules: to divide some number by a fraction, you simply "invert and multiply" or "multiply by the reciprocal of the fraction." But very few students understand why this works.

To help his sixth-grade students develop understanding of division by fractions, Mr. Harris plans several days of instruction. First he discusses with the students the fact that in this set of lessons they would encounter some new ideas about division. He writes on the board a simple division problem with whole numbers:

$$12 \div 4 = \underline{\quad\quad}$$

He asks the students to give an example of what the problem might mean. Students respond with "sharing" interpretations. For example, one student says, "If you share twelve pies among four people, how many pies will each get?" Mr. Harris reminds the students of another way to interpret a division problem, as making equal groups rather than sharing. He counts out twelve sheets of paper and tells the students that they will use them to make booklets. "We need four sheets for each booklet," he says. "How many booklets can we make with these twelve sheets of paper?" The answer is obvious to the students, and Mr. Harris models it by putting the paper into piles, or groups, with four in each, having the students do the subtraction to figure out how

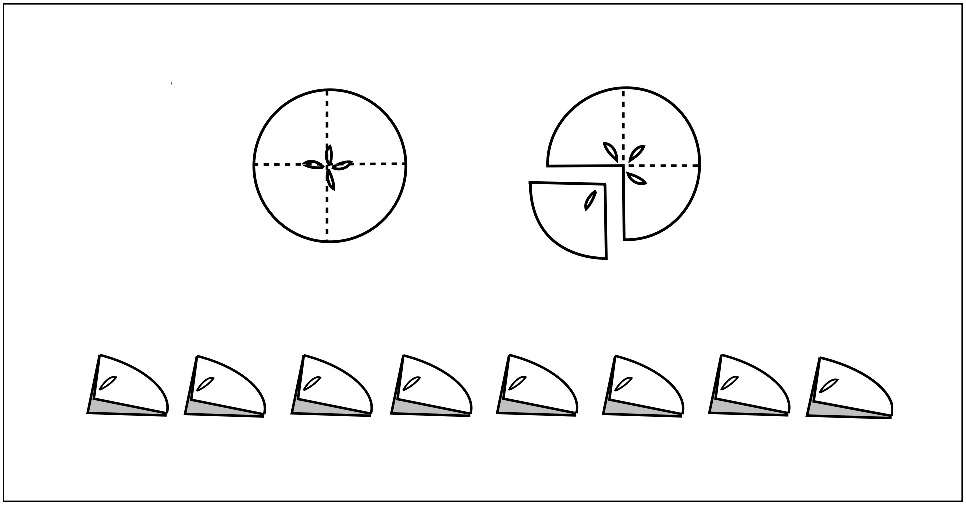

FIGURE 2–3 **Mr. Harris's illustration, showing how one-fourth can be subtracted from two pies eight times.**

many sheets of paper are left each time he makes a group of four. In this way, he reviews interpreting division as repeated subtraction.

Mr. Harris then writes another equation on the board:

$$2 \div \frac{1}{4} = \underline{\qquad}$$

He presents the conundrum of dividing two pies among one-fourth of a person. The students all agree that this does not make much sense to them. Then he gives another interpretation of the problem, as dividing the two pies into equally sized groups with one-fourth of a pie in each group. By drawing pictures, he illustrates the process of successively subtracting one-fourth of a pie from two pies to arrive at eight groups, thus showing how the answer of 8 is derived. (See Figure 2–3.)

Finally, Mr. Harris writes on the blackboard the following table of number sentences, in which 2 is divided by some unit fraction.

$2 \div \frac{1}{2} = ?$
$2 \div \frac{1}{3} = ?$
$2 \div \frac{1}{5} = ?$
$2 \div \frac{1}{6} = ?$
$2 \div \frac{1}{9} = ?$
$2 \div \frac{1}{20} = ?$
$2 \div \frac{1}{50} = ?$

Mr. Harris calls on different students to read each number sentence aloud using this format: "How many times can we subtract $\frac{1}{5}$ from two?" For example, "How many times can we subtract one-fiftieth from two?" He then asks students to talk together in groups of four to solve the entire set of number sentences. He gives them twenty minutes to do this as he circulates around the room, checking in with different groups.

Mr. Harris's use of small-group discussion

Mr. Harris thinks that the students need time to think about the answers to the problems. If he asks them to do so individually, he knows that students who are having difficulty could be stranded without help. Therefore, Mr. Harris uses the small-group format and asks students to work together on the problems. This provides support for the students, giving them a chance to talk with one another about their thinking. It keeps all of them engaged with the problems. And it allows Mr. Harris to circulate and check on the students' understanding. When he notices that most groups have written answers for all of the problems, Mr. Harris asks the students for their attention. Then, calling on students to report their answers, he fills in the chart. As students report, he revoices their answers and then reinforces the subtraction idea: "So you found that two divided by one-sixth is twelve, because you can subtract one-sixth from two . . . *twelve* times. Is that right?" Finally the table looks like this:

$2 \div \frac{1}{2} = 4$
$2 \div \frac{1}{3} = 6$
$2 \div \frac{1}{5} = 10$
$2 \div \frac{1}{6} = 12$
$2 \div \frac{1}{9} = 18$
$2 \div \frac{1}{20} = 40$
$2 \div \frac{1}{50} = 100$

1. Mr. H: OK, so now we've filled in the chart. Does anyone see a pattern?

Mr. Harris waits for thirty seconds. He asks again: "What pattern do you see?" The students are staring at the board, perusing the table. No hands are raised. Mr. Harris again uses "wait time" with determination, but then begins to think that perhaps this situation calls for something more drastic. He decides that the students need to voice their questions and externalize their thinking with one another first, before they tackle talking about this complex problem in the context of the whole class.

2. Mr. H: Turn and talk to your partner about the table. Do you see a pattern that relates the quotient to the first two numbers? See if you can come up with an answer that you and your partner agree on.

The room explodes in talk. Students had ideas about the pattern, but they were hesitant about voicing these developing ideas in front of the whole class. Many students are reluctant to speak up when they are not absolutely sure of themselves. However, if they are given the opportunity to talk to one other person, a peer, in an environment where only that one other person will be paying attention to their initial attempts to articulate their thoughts, they gain confidence and clarity with respect to their thinking about new concepts.

Mr. Harris's use of partner talk

At this point, let's consider one of the benefits of Mr. Harris's use of talk. Although many of the students may have noticed a pattern in the numbers listed in the table, they are unable or unwilling to describe it aloud at the start. Partner talk or "turn and talk" is especially effective when students reach an impasse—the teacher gives students a short time, perhaps a minute or two at the most, to put their thoughts into words with their nearest neighbor. As noted, often students are reluctant to voice their partially formed mathematical ideas in front of a whole-class group. Mr. Harris also has students who are learning English as a second language who may be hesitant for both linguistic and mathematical reasons. All students can benefit from the two minutes of partner talk, particularly when faced with a knotty new problem. This talk practice is simple, but it can have a profound impact on ensuring equitable participation in the classroom. While a few students may always be ready at a moment's notice to give an answer to anything, the whole class suffers if the teacher is unable to create the conditions for equal access for all students.

After the two minutes, Mr. Harris continues:

3. Mr. H: So let's hear what you have decided about the pattern in the chart. Verette?
4. Verette: Me and my partner think the pattern is that the answer is twice as much as the bottom number.
5. Mr. H: Let's hear from others. George.
6. George: We think you take the denominator and multiply it by the first number, and then you get the quotient.

7. Mr. H: Is the pattern that Verette found the same as the pattern that George found? Paula?
8. Paula: I think they are the same, because Verette said the bottom number and George said the denominator, and those are the same thing, and you multiply them by the first number, which is two.
9. Mr. H: Does the pattern work for every single case? Let's check each example. If we take the first number, two, and multiply it by the denominator in the first example, two, what do we get? Four. It works. Let's try the next one. Marcia, can you talk us through that one? [Mr. Harris continues through the entire chart, having individual students talk through the calculation for each one.]

Recognizing and understanding the teacher's moves

Notice how Mr. Harris uses the contribution of different students in a sequential fashion to build understanding for the entire group. He does not simply announce the pattern at the start. Nor does he correct or clarify the contribution of the first student, Verette. Instead of evaluating Verette's formulation he asks for another formulation from a different student. The next student, George, presents the same generalization but in different terms. Still Mr. Harris does not evaluate, but instead moves on to ask whether these two are the same. The third student, Paula, gives yet a third formulation of the same generalization in slightly different words, combining George's and Verette's contributions.

Mr. Harris's moves here are not necessarily the best or only way to handle this conversation. He could certainly have homed in on Verette's contribution and elicited more detail, as he has done many other times. But we offer this example as a way of showing how a skillful teacher can use the contributions of students to accomplish several goals at once. First, Mr. Harris is giving the students practice in putting their mathematical reasoning into words. Second, he is letting students who need more processing time hear several different versions of the target generalization. And third, he is modeling for students the practice of listening to one another, and conveying their obligation to try to make sense of one another's words.

As the lesson goes on, Mr. Harris gives the students another set of examples, in which the dividend for each problem is 3, not 2, and the divisor is still a unit fraction with a numerator of 1. He again asks them to look at the examples and find a pattern. And he again uses partner talk to give students time to see that once more the quotient equals the dividend times the denominator (e.g., $3 \div \frac{1}{5} = 15$).

All in all, Mr. Harris invokes partner talk nine times during the entire lesson. And the rewards are great. With a step-by-step, careful, and systematic approach, Mr. Harris gives plenty of time for students to process the current question with their partner, and he uses students' subsequent contributions to solidify everyone's understanding. This leaves the students ready to work the next day on generalizing still further, beyond division by unit fractions (e.g., $2 \div \frac{2}{5}$ or $3 \div \frac{2}{5}$). The computations in problems like these are more difficult: repeated subtraction of $\frac{2}{5}$ from 2 is not as straightforward as repeated subtraction of $\frac{1}{5}$ from 2. And while the answer to $2 \div \frac{2}{5}$ is a whole number, 5, as were all the answers in the earlier tables, the answer to $3 \div \frac{2}{5}$ isn't a whole number. If you subtract $\frac{2}{5}$ from 3 seven times, you are left with $\frac{1}{5}$, which isn't enough to subtract another $\frac{2}{5}$. It's only half of what's needed. The answer to $3 \div \frac{2}{5}$ is $7\frac{1}{2}$, which means that you can subtract $\frac{2}{5}$ from 3 seven and a half times. This is complex for students to grasp. Nevertheless, by continuing slowly and carefully, alternating among partner talk, small-group discussion, and whole-class discussion, and building on students' contributions, Mr. Harris was able eventually to lead students to adopt the strategy of multiplying the dividend by the reciprocal of the divisor. But in their case it would not simply be a computational trick, it would be undergirded by a real understanding of the mathematical relationships. In Mr. Harris's view, what the students accomplished would not have been possible without the extensive use of the talk moves and strategies we have been describing.

Integrating Talk and Content

Each of these teachers had taken the time to set up classroom norms for respectful and courteous discourse. They had also made it possible for all students to have equal access to participation. Their students are obligated to listen quietly and to speak up when they make a contribution. They are not allowed to express disrespect, and they follow standard turn-taking procedures.

These teachers rely on three talk formats: whole-class discussion, small-group discussion, and partner talk, using whole-class discussion and partner talk most often to get students to focus on mathematical reasoning, both their own and that of their classmates. Each of these teachers uses other talk formats as well, but in these lessons we paid special attention to these two because they facilitate the kind of mathematical thinking we are interested in supporting. Within the context of these talk formats, each teacher made skillful use of the talk moves introduced in this section.

But the talk formats and talk moves together still do not add up to a full and coherent lesson. As the teacher, you must integrate the tools of talk with the mathematical content, adjusting and refining both to the particulars of your own unique teaching situation. To help you, we suggest a three-part cycle that you will engage in again and again as you incorporate productive talk into your mathematics class.

1. Planning and projecting: creating a roadmap.
Your first step is to plan carefully for a specific lesson. You need to spend time identifying the important mathematical ideas and concepts you'll be talking about, as well as potential misconceptions or difficulties that students might have. You also need to plan which talk formats you will use, and how you will incorporate the specific talk moves we've introduced.

2. Improvising and responding: in the midst of the lesson.
As we all know, if you are actively engaging with students, even the most carefully planned lesson involves improvisation. You cannot be sure that things will go as you planned. And when you make extensive use of student talk, you introduce an element of uncertainty. Therefore, part of the cycle of introducing productive talk into your mathematics class necessarily involves improvisation and responding in the moment. And although you may not remember everything that happens in the lesson, you will aim to keep track of how your plans actually did or did not unfold as you expected.

3. Summarizing and solidifying: So where are we now?
During a talk-intensive lesson there will be moments when you and your students will feel overwhelmed: too much will have been said, and you will feel that you are losing focus. At these times, it's important to step back and review what has been said so far, and what the most significant points have been. Furthermore, after every such lesson, you will want to spend some time reflecting on what important mathematical ideas, conjectures, claims, and arguments have emerged during the class. When you return the next day, it will be important for you to present to the students a review of the most important aspects of the previous day's discourse. Talk-intensive lessons can be difficult to summarize, but as the teacher you will want to make sure that you take a few moments to review, clarify, and solidify the important points that have been made. This will also help you as you return to the "planning and projecting" part of the cycle.