Length

Introduction

Length is an attribute that students encounter naturally in their lives. Children want to know how much taller they have grown this year, how far they can kick a ball, and how tall a *Tyrannosaurus rex* would be. The lessons in this chapter build on the understandings that students bring to school, extending them to the concepts that are important for students to learn in grades 3 through 5.

Through these lessons students develop a sense of different units of length and acquire benchmarks for them, such as "a centimeter is about the length of my little fingernail" and "a meter is about the distance from my waist to the floor." Students determine reasonable units for measuring in different situations. They estimate lengths and distances and convert between shorter and longer units. Students create, make sense of, and use measurement tools with nonstandard and standard units. These lessons help students move beyond merely learning a procedure for using a ruler—"line up the end of the ruler with one end of the object, and match the end of the object with a number." Instead they help students understand how a ruler works and how the lines, spaces, and numbers on a ruler represent distance. Through these lessons students solve problems that help them better understand both the process of measurement and the world around them.

While students' home experiences typically revolve around the U.S. customary system of units, the rest of the world uses metric units, and students must use both systems of measurement in our increasingly global society. Therefore most of the lessons in this chapter can be adapted for either system of measurement.

Foot-Length Rulers

Overview

This lesson introduces students to the concept of linear measurement. They predict the width of the cafeteria using their own feet as units and then create foot-length rulers for

measuring it and other items in the cafeteria. During this lesson students learn about iteration (repeating a unit), the need for units to be the same size, the role of zero in a measuring device, and the meaning of the lines on a ruler.

Materials

- chart paper, about 3 feet by 4 feet, 2 sheets
- markers
- 12-inch rulers, 1 per pair of students
- color tiles, 6
- 6-foot strips of adding machine tape, 1 per pair of students (**Note:** you could also use 3- to 5-inch wide strips of paper 6 feet long.)

Vocabulary: distance, feet, foot length, inch, inverse relationship, iterate, measure, ruler, zero

Instructions

Measuring the Cafeteria with Feet

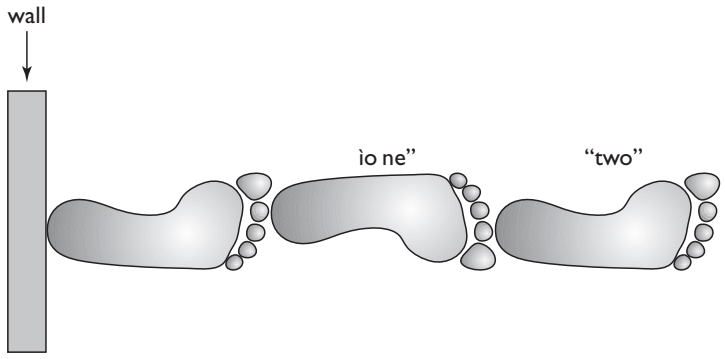
1. Prior to this lesson, tape a sheet of chart paper to the wall somewhere in the cafeteria where the students can gather and see it.

2. Explain to the students that they will measure the width of the cafeteria. Have them imagine that rulers have not been invented yet, and there is no such thing as inches or yards or meters. Tell them that a long time ago people used parts of the body to measure things. Ask them what parts of the body would be easiest to use to measure. They will likely suggest feet.

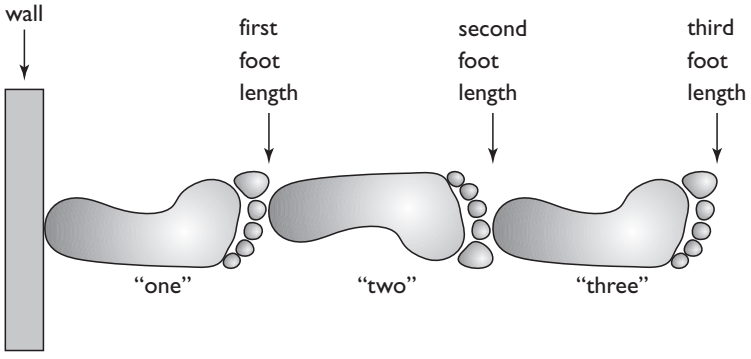
3. Now tell the students that they will use their feet to measure the distance from one side of the cafeteria to the other. Ask a student to demonstrate, with his shoes on, how he would begin. The student will most likely stand with the heel of one foot against the wall, then place the other foot right in front, with the heel of that foot touching the toes of the first foot. He will move forward, alternating feet in this way, counting as he walks across the room.

4. After a few steps, stop the volunteer and ask the class to explain why the student didn't begin measuring in the middle of the floor. (He has to cover the entire distance across the cafeteria.) Next ask the student to start over again so the class can look closely at what he is doing. Before the student begins, ask the others how they should count each time the student places a foot. Some students will think that the first foot counts as zero and the second foot

counts as one. These students are counting footsteps, not the length of each foot.



Others will understand that each foot length is counted: the first foot counts as one and the second as two.



Discuss the reasoning behind each perspective. Remind the students that in order to measure correctly, the feet must cover the entire distance from one side of the cafeteria to the other.

5. Have the student begin again and take three steps. As he steps, say, "First foot length, second foot length, third foot length." Ask the students where zero is (the wall) and where one is (the end of the first foot length). These questions can be confusing to students. They will have many chances to revisit the questions throughout this and other lessons.

6. Now have the class estimate how many foot lengths it will take for the student to cross the cafeteria. Have the student find the actual number of foot lengths and write his name and the number on the chart paper.

7. Have the students predict how many of their own feet wide the cafeteria is and discuss their reasoning. For example, many students will compare their foot with the volunteer's foot and use that information to estimate a larger or smaller number.

8. Now, have the students find the actual measurement using their own feet with the help of a partner. Ask the students to write their names and numbers on the chart.

9. When everyone is ready, discuss the numbers. Ask the students why there are different numbers when the distance did not change.

10. Circle one of the larger numbers. Ask the students what they know about that person's foot. Ask the students to explain why a smaller foot would result in a bigger number. Have different students explain this important inverse relationship in their own words, giving several students an opportunity to articulate it. Then circle one of the smaller numbers and again discuss what the students know about that person's foot.

Making a Foot-Length Ruler

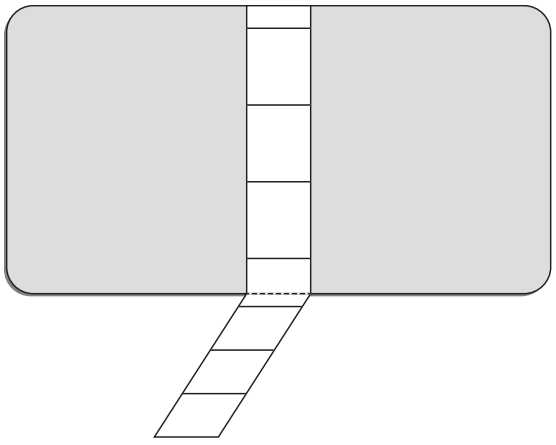
1. Tell the students that it takes a long time to measure by putting a foot down, then another, over and over again. Explain that a foot-length ruler would be faster to use and would be useful for measuring things where feet can't go, such as the height and width of a window.

2. Tell the students that they will work in pairs to make a foot-length ruler by marking the lengths of the feet of one of the students with lines on the adding machine tape or on a long, thin piece of paper. Ask whether it makes sense for both students' feet to be on one ruler. Discuss the importance of the units being the same size.

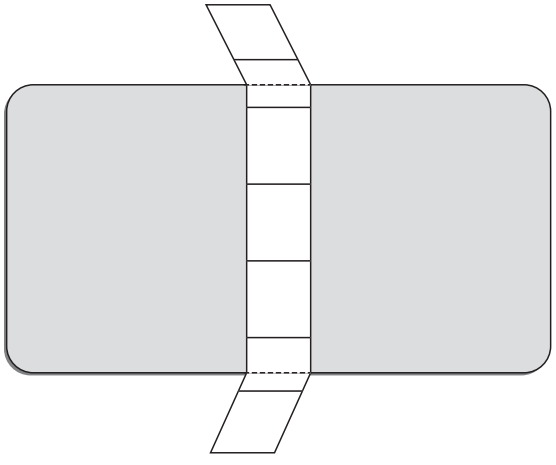
3. Pass out the paper and markers to the students. Explain that they will mark the end of each foot (however many will fit on the paper), and they will not use any numbers at all. As the students work, observe whether they begin with their heel at the end of the paper, whether they leave any spaces between footprints, and whether they are careful to put the next foot right against the last toe mark.

4. When everyone is ready, place three foot-length rulers on a cafeteria table and gather the students around them. Have the students compare them and tell what they notice. They will notice that the foot lengths on the different rulers are different sizes, but only slightly different. They also may notice that some rulers have different-size units on them. Discuss whether these rulers would be a good tool for measuring and have students explain their reasoning.

5. Some papers will have a small space between the last foot mark and the end of the paper. Select a foot-length ruler with an incomplete piece at the end. Ask the students to think about how they would use that ruler to measure the width of the cafeteria table, and choose one student to do this. As the student measures, have the other students explain why the volunteer is doing it that way. Discuss the following points with the students:
   - Ask, "Why didn't the volunteer begin counting with the small piece? Could she have lined up the end of the small piece with the edge of the table?" Discuss how, although that is not a standard way to use a ruler, it can be done. Talk about how they would estimate the length of the incomplete section.



- Discuss why the volunteer didn't place the ruler on the table so the edges hung off evenly and whether she could have done that. Discuss how, although that is not a standard way to use a ruler, it can be done. Talk about how that would work.



- Have the students explain what they are counting: the lines or the spaces. Have them explain in their own words that each space is the length of a foot and that each succeeding line represents another foot length.

6. Next remind the students that real rulers have lines with numbers on them. Ask the students to write numbers on their rulers.

7. Now select several foot-length rulers, some that have the numbers listed correctly and some that do not. Place them on the table and have the students compare them. The rulers that are done correctly will have the first line that completes a foot length past the edge labeled 1. Discuss what the number means: since the number is a label for the end of the foot length, it belongs at the first line.

8. Ask the students where 0 would be on their rulers. Discuss why it would be at the beginning edge of the paper.

9. Provide each pair of students with a standard 12-inch ruler. Have them look at it to see where the 0 would be. Then have the students look at the lines on their standard ruler to see what they mean. Place a standard 12-inch ruler on a table where all the students can see it. To show the repeating inches on the ruler, line up six color tiles against the ruler, beginning with the zero end. Point out that the lines on the ruler show where the color tiles end each time, just as the lines on the students' foot-length rulers show where their feet ended each time.

10. Return the students' attention to their foot-length rulers. Discuss how the students dealt with the little space left beyond their last foot length. Some may have torn it off, replicating how many commercial rulers are made. Some may have labeled the final edge with a fraction, or have used the fraction incorrectly. For example, a student may have incorrectly written:

$$1 \quad 2 \quad 3 \quad 4 \quad 5\frac{1}{2}$$

When you find students thinking incorrectly, ask them what each space means. Have them return to the beginning of the ruler and recount the spaces. In this case they would say, "One foot length, two foot lengths, three foot lengths, four foot lengths," and so on. Always push the students to see that the numeral represents how many foot lengths there are from the beginning of the paper.

11. Have the students write 0 on the start of their foot-length rulers and, if necessary, correct any other numbers.

Using the Foot-Length Rulers

1. Have the students help you select some things in the cafeteria that they could measure with their foot-length rulers. List these on a second sheet of chart paper. Include many things that are much longer than their rulers so that they will have to iterate their ruler. For example, you might list

the width of the door

the length of the stage

the length of the cafeteria

the length of a bench

the length of a table

the length of the display case

the height of the piano

the width of the bulletin board

the length of the window

the height of the menu sign

Tell the students to measure the items on the list and record their findings on a sheet of paper.

2. As the students work, observe how they use their rulers: whether they line them up correctly at the beginning, how they iterate the rulers, how they work with the fractional parts at the end of the rulers, and so on.

3. When the students have finished measuring most of the items, gather them for a discussion. Begin by asking them if they expect their answers to be the same and why. (Their foot lengths are different sizes, so the measurements will be different.) Ask them if they expect the answers to be very different and why. (The measurements should be fairly close.)

4. Now have students share their measurements for some of the items. As a group decide whether the measurements make sense. Also, ask them what they know about the foot of a person whose measurement is large. (As before, this indicates that the person's foot is small.)

5. Have the students share how they described several measurements that did not come out in whole foot lengths. Some students may use fractions of foot lengths, while others may use another body part to describe the piece, such as "a pinky."

How Big Is a Foot?

Overview

Rolf Myller's book *How Big Is a Foot?* provides the students with an occasion to think about the need for consistently sized units. The students listen to the story and then write a letter to the apprentice, explaining why the bed was the wrong size when the king used his large feet to measure the queen for a bed and then the apprentice used his smaller feet to build the bed. The students use the term *unit* and discuss the fact that, for units to have meaning, they have to be the same size.

Materials

- *How Big Is a Foot?* by Rolf Myller (1990)

Vocabulary: long, measure, unit, wide

Instructions

1. Show the students the cover of *How Big Is a Foot?* Have the students predict what the story might be about. If the students have already done the *Foot-Length Rulers* activity, encourage them to make a connection between the feet on the cover and the units they used in that activity.