Angles

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Introduction

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The teaching and learning of measurement with angles illustrate a shift in emphases that should occur in middle school—from focusing on individual concepts to comparing, contrasting, and connecting measurement topics. For example, students usually enter middle school with some knowledge about angles, usually exposure to categories of angles based on their measures. However, in middle school, through investigations with polygons, students discover that angle measures are directly connected to many of the properties of polygons and other shapes. Similarly, they build upon their knowledge of angles and the Triangle Inequality Theorem to discover the relationships expressed by the Pythagorean Theorem and applying this new knowledge can, in turn, open the whole world of irrational numbers for them in meaningful ways. (See *Pythagoras Plus* on page 158 and *That's Irrational!* on page 16.)

Angle measurement and learning to use the protractor are often introduced in middle school. Understanding the attribute of angle size and being able to estimate the measure of angles influence the success students have in using protractors, in making scale drawings, in finding the height of an object too tall to measure directly, and in making connections among measurement topics.

The typical definition of an angle—the union of two rays with a common end point—encourages students to think of an angle as a static construct and assume that the length of the rays determines an angle's size. The lessons in this chapter take an alternative approach. They focus on an angle as a rotation of one or both rays around a common end point. From this perspective, students view angle size on a continuum; that is, they understand that as one ray is rotated farther from the other, the measure of the angle increases.

The lessons use a variety of techniques that help students learn to use a protractor correctly. In addition to describing angles as being formed by rotations of rays, students examine angles with many different orientations. They use interlocking paper plates (see *Building Referents for the Measure of Angles* on page 23) to establish mental referents for common angles and patty paper to investigate angles formed by intersecting and parallel lines (see *Paper Angles* on page 29). Other lessons target angles in polygons (see

What's My Angle? on page 26) and angles in circles (see *Angles in the Round* on page 34) and encourage students to use patterns and reasoning to form conclusions.

Communication of mathematical thinking as well as at least one idea for extending the activity are built into each lesson.

Building Referents for the Measure of Angles

Related Topic: estimation

Overview
Students make an angle measurement device from paper plates. They then use it to establish angle referents. They use their referents and the device to create angles of any size.

Materials
❖ paper or foam plates, 1 each of two different colors per student
❖ rulers, 1 per pair of students
❖ protractors, 1 per student
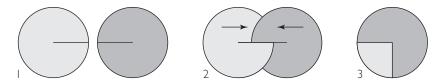❖ board protractor, transparent protractor, or transparency of a protractor

Vocabulary: acute angle, angle, degree, obtuse angle, referent, reflex angle, right angle, scale, straight angle
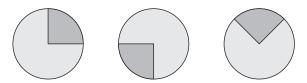
Instructions

1. Demonstrate to students the importance of estimation in measuring angles. You might do this by using a semicircular board protractor or overhead transparency of a protractor with two scales going in opposite directions. Draw a 60-degree angle and point out how a person who fails to estimate could easily read the angle as 120 degrees. You may also want to have students list activities and/or professions for which avoiding mistakes in angle measurement could be very important. Such a list could include the following examples:

architecture

astronomy

carpentry

cutting cloth and sewing quilt pieces

geography, including map reading

golfing, such as doglegs in fairways and
    angle of putts

graphic design

photography

pool or billiards

sailing

surveying

2. Tell students they will make and then use a device to help them estimate the measures of angles. Give each student two paper or foam plates of different colors. Have them use a ruler to measure and draw a radius in each plate. Then have them cut along each radius and fit the two plates together as shown below.

3. Have a volunteer define a *right angle*. Be sure the student mentions that a right angle contains 90 degrees and that it is shaped like a "perfect corner" or a "square corner." Suppose one of the plates is red. Ask students to hold one plate steady and rotate the other to show you a red angle that is approximately 90 degrees. Then have all the students hold their device facing you, so that you can verify that everyone has correctly shown the angle. Next have students rotate the device so that the 90-degree angle faces several different directions, as in the following examples.

Point out that a 90-degree angle is one that every student should have as a referent in her mind's eye, an image to which other angles can be compared.

4. Ask students to hold their angle device in their laps and rotate the plates to show you a red angle that is approximately 30 degrees. Again have all students hold their devices facing you, so that you can see how each student has shown the angle.

5. Without commenting on which students are showing good estimates, call on a student who has a good estimate to show the angle to the class and to explain how he decided how large a 30-degree angle is and how it should look. Often, students report that they know that a 30-degree angle is one-third the size of a 90-degree angle, so they make the angle one-third the size of the right angle they already made. Continue to call on students until you have elicited all strategies used by the class. If needed, give one of your own. Tell students that a 30-degree angle is another good referent to establish in their minds. Have students rotate their devices so the 30-degree angle faces several different directions.

6. Repeat the procedure described in Instruction 4 to have students show their estimates for an angle of 120 degrees. Point out that a 120-degree angle is a 90-degree angle plus a 30-degree angle so that they can combine the two referents they have discussed to estimate this new angle.

7. Help students establish other referents, such as 45 degrees and 180 degrees, and then ask them to estimate more challenging angles for which they can use the referents you have already discussed. For example, a 150-degree angle can be thought of as a 180-degree angle minus a 30-degree angle. Continue to have students explain their estimating strategies and rotate the angle devices to see the angles in different positions.

8. If appropriate, work with reflex angles in addition to those that are less than or equal to 180 degrees.

Notes to the Teacher

This activity helps students establish mental referents for angle measures. Such referents help students make good estimates and avoid common errors, such as reading a protractor incorrectly.

When you use this activity with your students for the first time, you may find the following comments useful:

❧ Building a bank of angle referents takes time. While this lesson helps students establish a good base, it's best to repeat the experience many times throughout the school year to firmly establish the visual benchmarks for your students.

❧ If you are able to find foam plates with thirty-six indentations around the circumference, students will eventually figure out that each indentation is equivalent to 10 degrees. Since this lesson is about learning to estimate the measure of angles by using a stored mental image of certain referent angles, these plates are helpful because students can create quite accurate angles if they use the indentations as guides. Alternatively, you may want to avoid using this "key" to force students to make their own estimates, even if they are not likely to be as accurate.

Extensions

❧ Ask students to use their angle device to show angles such as the complement of a 60-degree angle, the supplement of a 30-degree angle, or the third angle in a triangle if the other two angles measure 90 degrees and 45 degrees.

❧ The device can also be used with other topics. Ask students to show the following:
- decimals that are less than or equal to one
- percents that are less than or equal to 100 percent
- fractional equivalents for decimals less than or equal to one or percents less than or equal to 100 percent
- fractions that represent such ratios as 1 foot to 1 yard or 5 millimeters to 1 centimeter

Note: In many of these cases, the foam plates or other plates marked off in 10-degree arc angles work best if you are looking for an exact answer instead of an estimate.

What's My Angle?

Related Topics: algebraic thinking, exterior angles of polygons, interior angles of polygons

Overview

With a straightedge, students draw several different kinds of polygons. Then they use protractors to measure the interior and exterior angles of their shapes. They make generalizations about the measures and sums of such angles from the patterns they find in their data.

Materials

- *What's My Angle?* recording sheets, 1 per student (see Blackline Masters)
- straightedges, 1 per student
- protractors, 1 per student
- optional: board protractor or clear plastic protractor to use on an overhead projector

Vocabulary: decagon, diagonal, exterior angle, heptagon, hexagon, interior angle, n-gon, nonadjacent, octagon, pentagon, polygon, quadrangle, quadrilateral, septagon, 36-gon, triangle, vertex

Prerequisite Skills and Concepts

Students should know how to measure angles using a protractor. One set of online directions for this process can be found at www.ehow.com/how_12928_protractor.html and an interactive protractor is located at www.teachersfirst.com/getsource.cfm?id=6382.

Instructions

1. Ask a volunteer to define *acute angle*. Clarify the concept by using a board protractor to draw several angles on the board and asking students to identify each one as an example or a nonexample of an acute angle. Then ask each student to draw and measure an acute angle using a straightedge and a protractor. Have partners check each other's work. Repeat with another acute angle, if needed.

2. Repeat the process in Instruction 1 with obtuse angles.