CHAPTER 1

# What Is a Classroom Number Talk?

## Rationale for Number Talks

Recently, during a visit to a second-grade classroom, I watched Melanie subtract 7 from 13. She had written the problem vertically on her paper and began solving it using the standard U.S. algorithm for subtraction.

$$\begin{array}{r} {\scriptstyle 0\,13} \\ \cancel{13} \\ -\ \ 7 \\ \hline 6 \end{array}$$

I asked Melanie to share her thinking about this problem. She said, “I couldn’t take seven from three so I borrowed ten. And I made the one a zero and the three became thirteen. And thirteen minus seven is six.”

When asked why she chose to solve the problem this way, Melanie replied, “That’s just how you do it when the bottom number is bigger than the top.”

In a third-grade classroom, I also observed students using the standard U.S. algorithm to solve the problem 328 − 69.

$$\begin{array}{r} {\scriptstyle 2\,11\,18} \\ \cancel{328} \\ -\ \ 69 \\ \hline 259 \end{array}$$

I was curious about their understanding of this process and posed, “When you crossed out the numbers in three hundred twenty-eight and wrote different numbers above it, did you still have an amount that is the same as in three hundred twenty-eight?" The students looked puzzled by this question and collectively replied, "No, you have a different amount, because you have a two, an eleven, and an eighteen. That's why you can minus the six and the nine." Their struggle to make the connection between decomposing or breaking apart 328 into 200 + 110 + 18 was an indication that they did not fully understand the method they had been taught.

Like Melanie and this group of third graders, our classrooms are filled with students and adults who think of mathematics as rules and procedures to memorize without understanding the numerical relationships that provide the foundation for these rules. The teaching of mathematics has been viewed as a discrete set of rules and procedures to be implemented with speed and accuracy but without necessarily understanding the mathematical logic. For some people, learning mathematics as procedures has been successful; but for the majority of our nation, knowledge of mathematical rules has not allowed them to use math confidently in their daily lives. With almost two-thirds of the nation's adult population fearful of mathematics, they have simply said "No" to mathematics and closed the doors to careers that require higher math (Burns 1998).

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Today's students often elect not to pursue more complex mathematics courses and careers that require higher math after negative experiences with earlier mathematics and algebra. Their foundation based on memorization crumbles when they are called to generalize arithmetic relationships in algebra courses. Robert Moses, an avid proponent for equity in mathematics, refers to algebra as a gatekeeper with far-reaching effects on an individual's ability to succeed. "So algebra, once solely in place as the gatekeeper for higher math and the priesthood who gained access to it, now is the gatekeeper for citizenship; and people who don't have it are like the people who couldn't read and write in the industrial age" (Moses and Cobb 2001, 14).

While our current understanding and approaches to mathematics may have been sufficient during earlier time periods in the United States, today's information age requires students and adults to develop a deeper understanding of mathematics. Our students must have the ability to reason about quantitative information, possess number sense, and check for the reasonableness of solutions and answers. We need citizens who are able to discern whether numbers make sense and are applicable to specific situations and who are capable of communicating solutions to problems. Today's mathematics curricula and instruction must focus on preparing students to be mathematically proficient and compute accurately, efficiently, and flexibly.

What does it mean to compute with accuracy, efficiency, and flexibility? *Accuracy* denotes the ability to produce an accurate answer; *efficiency* refers to the ability to choose an appropriate, expedient strategy for a specific computation problem; and *flexibility* means the ability to use number relationships with ease in computation. Take as an example the problem $49 \times 5$. A student exhibits these qualities in her thinking if she can use the relationship between 49 and 50 to think about the problem as $50 \times 5$ and then subtract one group of 5 to arrive at the answer of 245. If our goal is to create students who meet the above criteria, we must provide opportunities for them to grapple with number relationships, apply these relationships to computation strategies, and discuss and analyze their reasoning.

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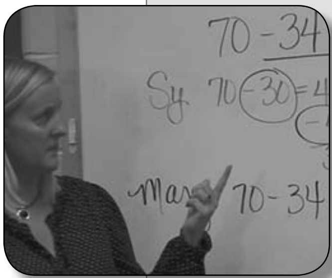
The introduction of number talks is a pivotal vehicle for developing efficient, flexible, and accurate computation strategies that build upon the key foundational ideas of mathematics such as composition and decomposition of numbers, our system of tens, and the application of properties. Classroom conversations and discussions around purposefully crafted computation problems are at the very core of number talks. These are opportunities for the class to come together to share their mathematical thinking. The problems in a number talk are designed to elicit specific strategies that focus on number relationships and number theory. Students are presented with problems in either a whole- or small-group setting and are expected to learn to mentally solve them accurately, efficiently, and flexibly. By sharing and defending their solutions and strategies, students are provided with opportunities to collectively reason about numbers while building connections to key conceptual ideas in mathematics. A typical classroom number talk can be conducted in five to fifteen minutes.

## A Number Talk: Key Components

Recently, I had the opportunity to visit Ms. Morton's third-grade classroom where number talks are an integral component of math

## Classroom Link: Subtraction: 70 – 34
Classroom Clip 3.4




Before you watch the third-grade number talk for 70 – 34, think about how you would mentally solve this problem.

As you watch the third-grade number talk for 70 – 34, consider

1. How are students using number relationships to solve the problem?
2. How would you describe the classroom community and environment?
3. Which strategies demonstrate accuracy, efficiency, and flexibility?
4. How are the students' strategies similar or different from your strategy?

For commentary on the above, see Appendix A: Author's Video Reflections.

instruction. The third graders are sitting in the front of the classroom while Ms. Morton writes the problem *102 − 76* horizontally on the chalkboard. Within seconds students begin to place their thumbs on their chests, signaling that they have mentally arrived at a solution. Ms. Morton asks, "What answer did you get?" Students raise their hands, and the teacher records all student answers on the chalkboard: *26*, *126*. "Who would like to defend their answer?" she poses.

Sylvia says, "I want to defend twenty-six. I used an adding-up strategy. I added four to seventy-six to make eighty, and then added on twenty to get to one hundred, plus two more to get to one hundred two. So the four plus twenty plus two gave me twenty-six."

As Sylvia shares her strategy, Ms. Morton uses an open number line to model Sylvia's thinking. (See Figure 1–1.)

"Did anyone think about this problem in a different way?" questions Ms. Morton.

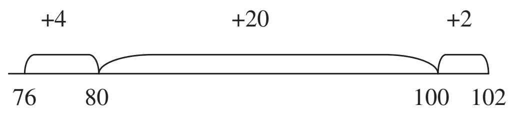



Figure 1–1 *Adding-Up Strategy for 102 − 76*

"I got twenty-six, too, but I did it another way. I added on four to each number to change the problem to one hundred six minus eighty, and I knew that one hundred six minus eighty was twenty-six. And that was easy to solve in my head!" shares Brendan.

$$\begin{array}{rcr} 102 + 4 = & & 106 \\ \underline{-\ \ 76} + 4 = & & \underline{-\ 80} \\ & & 26 \end{array}$$

"Do you follow Brendan's strategy? Can we prove it using the number line?" asks Ms. Morton.

"I do," offers Carla. "I think it will always work, because you're keeping the space between the numbers the same."

"What do you mean?" questions Marquez.

"Well, let's say you have the problem five minus three. We know that's going to be two, right? Well, if you add the same amount to both the three and the five, you will still get two when you subtract."

"Let's test Carla's idea," proposes Sierra.

Carla shares, "I'll use the problem five minus three again. If we add one to the five and one to the three, the problem would become six minus four, which still equals two. If we added two to the five and two to the three, then we'd get the problem seven minus five and this still equals two. We could even add three to each of the numbers to make eight minus six, and it still comes out as two!" (See Figure 1–2.)

Melinda jumps up to the number line on the board. "Oh, I get it! You're moving both of the numbers up the same amount on the number line, so the distance between them stays the same! As long

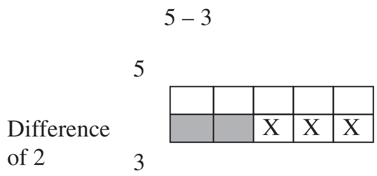
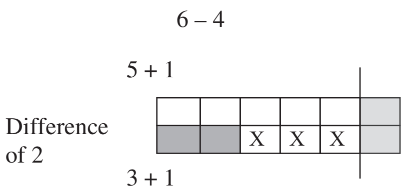
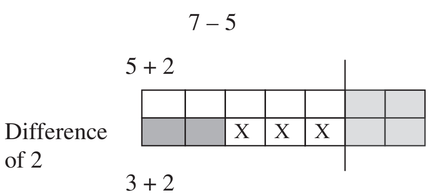
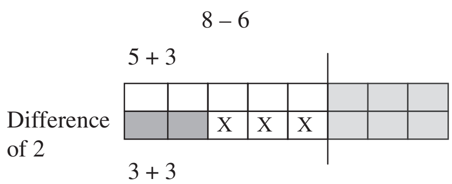
| | |
|---|---|
|  | *The original problem of 5 – 3 is represented with tiles.*<br><br>*The difference between 5 – 3 is 2 as indicated by the shaded portion.* |
|  | *The problem becomes 6 – 4 by adding 1 to the 5 and 1 to the 3.*<br><br>*The difference remains 2 as indicated by the shaded portion.* |
|  | *The problem becomes 7 – 5 by adding 2 to the 5 and 2 to the 3.*<br><br>*The difference remains 2 as indicated by the shaded portion.* |
|  | *The problem becomes 8 – 6 by adding 3 to the 5 and 3 to the 3.*<br><br>*The difference remains 2 as indicated by the shaded portion* |

Figure 1–2 *Square Tiles to Represent the Constant Difference Strategy for 5 – 3*

as you add the same amount to each number, the difference will stay the same!"

Marquez says, "I see now, but why did you add four to each number in the problem one hundred two minus seventy-six?"

"Because I was trying to get the seventy-six to an even ten of eighty," replies Brendan.

"So, this is a strategy we might want to investigate a little further today after our number talk," responds Ms. Morton. "I noticed that we have two strategies that support the answer of twenty-six for our original problem of one hundred two minus seventy-six. We also have the answer of one hundred twenty-six. Would someone share their thinking about this answer?"

"I don't think it's reasonable," offers Conner, "because seventy-six plus just thirty more puts you over one hundred two."

"Agree," several students chime in while nodding their heads in agreement with Conner.

Robert speaks up, "I disagree with myself, but I'm not sure why my strategy didn't work. I put the problem up and down in my head. I couldn't take six from two, so I went next door to borrow a ten. So I crossed out the zero and made it a nine and made the two a twelve. Twelve minus six is six and nine minus seven is two, and then I brought down the one. That's how I got the answer one hundred twenty-six."

$$\begin{array}{r} {\scriptstyle 9\;12} \\ 1\cancel{0}\cancel{2} \\ -\ 76 \\ \hline 126 \end{array}$$

"Oh, I think I see what you're trying to do," offers Amelia. "Are you trying to borrow ten from the one hundred and give it to the two? It's really like breaking up one hundred into ninety and ten or renaming one hundred two to ninety plus twelve."

As an upper-grade elementary teacher, my initial classroom experiences were full of Roberts, and I grew to predict with certainty the errors that would come with regrouping in addition, borrowing in subtraction, multiplying with multidigit numbers, and applying the steps of long division. We would study and practice the procedures for computing with these operations, but students continued to produce answers that were far from reasonable. My students were trying to remember the rules and apply them, but for the majority of them, math simply did not make sense.

For the students in Ms. Morton's classroom, math is about making sense of numerical relationships. They understand place value and can apply this concept in their computation. They can compose and decompose numbers and use these relationships to structure efficient, flexible, and accurate strategies. They know there is a strong connection between addition and subtraction, and they build upon this relationship with adding up on the number line. They understand that subtraction involves finding the difference between quantities. Students are able to defend their thinking and support it with mathematical reasoning and proof.

We can extract several key components of number talks from Ms. Morton's classroom. While each of these components will be discussed separately, they are interdependent and tightly interwoven:

*Key Components of Number Talks*

1. Classroom environment and community
2. Classroom discussions
3. The teacher's role
4. The role of mental math
5. Purposeful computation problems

## Classroom Environment and Community

Building a cohesive classroom community is essential for creating a safe, risk-free environment for effective number talks. Students should be comfortable in offering responses for discussion, questioning themselves and their peers, and investigating new strategies. The culture of the classroom should be one of acceptance based on a common quest for learning and understanding. Did you notice how Robert was willing to risk being incorrect with his peers, and how his fellow classmates were committed to working toward a collective understanding?

It takes time to establish a community of learners built on mutual respect, but if you consistently set this expectation from the beginning, students will respond.

## Classroom Discussions

During a number talk, the teacher writes a problem on the board and gives the students time to solve the problem mentally. Students start with their fists held to their chests and indicate when they are ready with a solution by quietly raising a thumb. Once students have found an answer, they are encouraged to continue finding efficient strategies while others are thinking. They indicate that they have found other approaches by raising another finger for each solution. This quiet form of acknowledgement allows time for students to think, while the process continues to challenge those who already have an answer. When most of the students have indicated they have a solution and strategy, the teacher calls for answers. All answers—correct and incorrect—are recorded on the board for students to consider.

Students are now given the opportunity to share their strategies and justifications with their peers. The benefits of sharing and discussing computation strategies are highlighted as follows:

***Benefits of Sharing and Discussing Computation Strategies***

Students have the opportunity to:

1. Clarify their own thinking.
2. Consider and test other strategies to see if they are mathematically logical.
3. Investigate and apply mathematical relationships.
4. Build a repertoire of efficient strategies.
5. Make decisions about choosing efficient strategies for specific problems.

Do children come up with incorrect answers in number talks? Absolutely. However, students are asked to defend or justify their answers to prove their thinking to their peers. In number talk classrooms, students have a sense of shared authority in determining whether an answer is accurate. The teacher is not the ultimate authority, and students are expected to think carefully about the solutions and strategies presented.

In number talks, wrong answers are used as opportunities to unearth misconceptions and for students to investigate their thinking and learn from their mistakes. In a number talk classroom, mistakes play an important role in learning. They provide opportunities to question and analyze thinking, bring misconceptions to the forefront, and solidify understanding. Helping students realize that mistakes are opportunities for learning is an important cornerstone in building a learning community.

## The Teacher's Role

As educators, we are accustomed to assuming the roles of telling and explaining. Teaching by telling is the method most of us experienced as students, and we have continued to emulate this model in our own practice. Since a goal of number talks is to help students make sense of mathematics by building upon mathematical relationships, our role must shift from being the sole authority in imparting information and confirming correct answers to assuming the interrelated roles of facilitator, questioner, listener, and learner.

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Since the heart of number talks is classroom conversations, it is appropriate for the teacher to move into the role of facilitator. Keeping the discussion focused on the important mathematics and helping students learn to structure their comments and wonderings during a number talk is essential to ensure that the conversation flows in a meaningful and natural manner. As a facilitator, you must guide the students to ponder and discuss examples that build upon your purposes. By posing such questions as "How does Joey's strategy connect to the ideas in Renee's strategy?" you are leading the conversations to build on meaningful mathematics.

When I began listening to my students' thinking, I realized I had much to learn about students' natural intuitions regarding numbers. Instead of only concentrating on a final, correct answer and one procedure for getting there, I began to broaden my scope to engage in listening to and learning about students' natural thinking through asking open-ended questions. My initial focus of "What answer did you get?" shifted to include "*How* did you get your answer?" I had made an assumption that all of my students solved their computation problems in the way I had taught them. While initially I was only interested in finding out their final answers to a problem such as 16 + 17, my focus broadened to learning about *how* they arrived at their answers and *why*. Did they use the doubles of 15 + 15 to solve 16 + 17, or did they combine the 10s to make 20 and then add the 13 from the total in the ones column? This was information I did not know and could use to help them look at how numbers are interrelated in different operations. By changing my question from "What answer did you get?" to "How did you solve this problem?" I was able to understand how they were making sense of mathematics.

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## The Role of Mental Math

Mental computation is a key component of number talks because it encourages students to build on number relationships to solve problems instead of relying on memorized procedures. One of the purposes of a number talk is for the students to focus on number relationships and use these relationships to develop efficient, flexible strategies with accuracy. When students approach problems without paper and pencil, they are encouraged to rely on what they know and understand about the numbers and how they are interrelated. Mental computation causes them to be efficient with the numbers to avoid holding numerous quantities in their heads.

An example that illustrates this idea is a common strategy for solving the problem $12 \times 49$. If students think about multiplying by a multiple of 10, they often change the 49 to 50 and multiply $12 \times 50$ for a product of 600. Since they multiplied by one extra group of 12, they subtract 12 for a final answer of 588. This strategy not only exhibits flexibility with the numbers, but is an efficient strategy that produces an accurate answer.

Another rationale for mental computation is to help strengthen students' understanding of place value. By looking at numbers as whole quantities instead of discrete columns of digits, students have to use their knowledge of place value. During initial number talks, problems are often written in a horizontal format to encourage the student's thinking in this realm. As students become accustomed to reasoning about the magnitude of numbers and utilizing place value in their strategies, the teacher may present problems both horizontally and vertically.

A problem such as $199 + 199$ helps illustrate this reasoning. By writing this problem horizontally, you encourage a student to think about and utilize the value of the entire number. A student with a strong sense of number and place value should be able to consider that 199 is close to 200; therefore, 200 + 200 is 400 minus the two extra 1s for a final answer of 398:

$$\begin{array}{r}
199 + 199 \\
\underline{\phantom{19}1} + \underline{\phantom{19}1} \\
200 + 200 = 400 \\
- \underline{\phantom{40}2} \\
398
\end{array}$$

Recording this same problem in a vertical format can encourage students to ignore the magnitude of each digit and its place value. A student who sees each column as a column of ones would not be using real place values in the numbers if they are thinking about 9 + 9, 9 + 9, and 1 + 1:

$$\begin{array}{r|c|c}
1 & 1 & \\
1 & 9 & 9 \\
+\ \underline{1} & \underline{9} & \underline{9} \\
3 & 9 & 8
\end{array}$$

## Purposeful Computation Problems

Crafting problems that guide students to focus on mathematical relationships is an essential part of number talks that is used to build mathematical understanding and knowledge. The teacher's goals and purposes for the number talk should determine the numbers and operations that are chosen. Careful planning before the number talk is necessary to design "just right" problems for students.

For example, if the goal is to help students use strategies that build upon using tens, starting with numbers multiplied by 10 followed by problems with 9 in the ones column creates a situation where this type of strategy is important. Problems such as 20 × 4, 19 × 4, 30 × 3, 29 × 3, 40 × 6, and 39 × 6 lend themselves to strategies where students use tens as friendly or landmark numbers. In the problem 19 × 4, the goal would be for students to think about 20 × 4 and subtract 4 from the product of 80, because they added on one extra group of 4. The other problems sets are designed to elicit a similar approach. In later number talks the teacher would start with a number with 9 in the ones column without starting with the multiple of 10.

Does this mean that given a well-crafted series of problems, students will always develop strategies that align with the teacher's purpose? No. Numerous strategies exist for any given problem; however, specific types of problems typically elicit certain strategies. Take, for instance, the same $19 \times 4$ problem that was crafted to target students' thinking using tens. Students could approach this problem in a variety of ways including, but not limited to, the following:

- Break the 19 into $10 + 9$; then multiply $10 \times 4$ and $9 \times 4$ and combine these products.
- Break the 4 into $2 + 2$; then multiply $2 \times 19$ and $2 \times 19$ and combine these products.
- Add $19 + 19 + 19 + 19$, or add 4 nineteen times.

However, a mixture of random problems such as $39 \times 5$, $65 - 18$, and $148 + 324$ do not lend themselves to a common strategy. While this series of problems may be used as practice for mental computation, it does not initiate a common focus for a number talk discussion.

## Summary

In this chapter we have looked at how number talks can be a purposeful vehicle for

1. making sense of mathematics;
2. developing efficient computation strategies;
3. communicating mathematically; and
4. reasoning and proving solutions.

In the following chapters, we will investigate how to design purposeful number talks with addition, subtraction, multiplication, and division while building on foundational concepts in arithmetic.