

# LESSON 1

## WALKING TRIP

### Common Core State Standards

Expressions and Equations 6.EE

Reason about and solve one-variable equations and inequalities.

5. Understand solving an equation or inequality as a process of answering a question. Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
7. Solve real-world and mathematical problems by writing and solving equations of the form  $x + p = q$  and  $px = q$  for cases in which  $p$ ,  $q$ , and  $x$  are all nonnegative rational numbers.

Represent and analyze quantitative relationships between dependent and independent variables.

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate those to the equation.

### LESSON GOALS

- Introduce patterns of change between variables.

### LESSON INTRODUCTION

This lesson is about rate.

Ask students to give examples of rates and make a list on the board of the rates they share. In this lesson, students may use tables, graphs, or equations to solve the problems.

A **rate** is a ratio of two measurements.

**For students to get a complete understanding of rates and the proportional relationship involved, it is important to make connections between the different representations used by students.**

For one question in the Student Recording Page, students are asked to find their own walking rate. Upon completion of the lesson, you can have students share their strategies for finding how long it would take them to walk 2 miles. Once again students can approach this situation using a variety of different rates and a variety of different strategies. Discuss with students the use of proportional reasoning in all of the strategies.

Students should realize as they progress through this lesson that a proportional relationship is graphed as a straight line that passes through the origin.

## TEACHING SUGGESTIONS

Within the bulleted text below are suggestions on how to begin the lesson, introduce essential vocabulary, and question students so that they are prepared to complete the Student Recording Page on their own or with minimal assistance.

- If I told you my walking rate was 0.75, would that be enough information to predict how far I could walk in 1 hour?
- What additional information would you need?
- Discuss walking rates. What is a walking rate?
- How could you find a walking rate? What two variables do you need to find a walking rate?
- How are walking rates recorded?

Have a discussion about how some rates are reported.

- In this lesson, you will be asked to find your own walking rate. What do you think you will need to find your walking rate?

Discuss students' ideas briefly. Leave the discussion open enough to allow students to decide how they will collect and record their walking rates.

Introduce the problem to students.

- Does Yvonne and Lisa's walking rate seem reasonable?
- When students use rates in the Student Recording Page, instruct them to always include labels.
- Instruct students to pay attention to the scales used when graphing so the data does not get distorted.

### NCTM Correlation

#### Algebra

Understand patterns, relations, and functions.

*Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and when possible, symbolic rules.*

Represent and analyze mathematical situations and structures using algebraic symbols.

*Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships.*

### Materials

Student Recording Page 2.1

### Vocabulary

Rate

## STUDENT ENGAGEMENT

Students are to complete Student Recording Page 2.1. When students begin work on Question 7, have them work in groups of three or four. They are finding their own walking rates and will need access to a hallway or breezeway to collect their times and distances data.

As students work on this lesson, ask questions to check for understanding. Ask questions about the relationships they are using. If students are using rates, ask them to define the rates using labels.

## QUESTIONS FOR STRUGGLING STUDENTS

What is Yvonne and Lisa's walking rate? Explain.

*They walk 1.5 meters per second. For every second that passes, they have walked 1.5 meters.*

Sketch a graph that represents Yvonne and Lisa's walking distance. How far have they walked after 1 second? 3 seconds? 10 seconds? How did you determine these distances?

*For every second that passed, the total distance increased 1.5 meters.*

How can you organize these time and distance amounts as ordered pairs?

*(time, distance)*

Where is 9 meters represented on your model? How long will it take Yvonne and Lisa to have walked that far? Explain.

*On the sketch of the graph, 9 meters would be the ordered pair (9, 13.5).*

What ordered pair of time and distance values represents the start of the walk?

*(0, 0)*

## QUESTIONS FOR STRUGGLING STUDENTS (CONTINUED)

What does it mean when the problem states, “they walk at a constant rate of 1.5 meters per second?”

*They walk at a steady rate in a proportional relationship.*

What does their walking rate mean?

*the distance a person travels in a given amount of time*

What units are used in their walking rate?

*meters and seconds*

If you know how far they walk in a second, how can you find out how far they walk in a minute? Hour?

*Multiply the distance by 60 because there are 60 seconds in a minute. Multiply that answer by 60 because there are 60 minutes in an hour.*

How do you think Lisa determined the constant walking rate?

*She walked for a certain amount of time and found how far she had walked.*

If you had a timer and a tape measure, how could you determine your walking rate?

*Measure a distance on the floor and time how long it takes to walk that distance.*

How long do you think it will take you to walk from here to there (e.g. one end of chalk board to the other) walking at your usual pace? How can you check your prediction?

*about 1 second; find the walking rate*

## QUESTIONS FOR STRUGGLING STUDENTS (CONTINUED)

How far do you think you should walk to establish an accurate walking rate?

*Possible response: Walk for at least 10 seconds or 10 meters.*

What units will you use to find your walking rate?

*meters/second or seconds/meter*

## QUESTIONS FOR STUDENTS ON TASK

Name two unit rates in this problem.

*1.5 meters per second, 1,600 meters in a mile*

How does knowing how many meters are in a  $\frac{1}{2}$  mile help you?

*The information could be used to find the number of meters in a mile.*

How many seconds are in an hour?

*3,600*

If you know meters per hour, how can you find meters per minute?

*Divide the number of meters by 60, because there are 60 minutes in 1 hour.*

What are some rates that might help you answer the questions?

*seconds/minute, minutes/hour, meters/hour, miles/hour*

What are the variables in this situation?

*time and distance*

Which variable is the independent variable and why?

*time, because distance is dependent on time*

## QUESTIONS FOR STUDENTS ON TASK (CONTINUED)

What patterns of change do you see?

Students may suggest: The longer (time) you walk, the longer distance travelled. For every hour that passes, Yvonne and Lisa walk 5,400 meters.

How does the distance for 3 hours compare to the distance at 6 hours? 9 hours?

The distance at 6 hours is double the distance at 3 hours, and the distance at 9 hours is triple the distance at 3 hours.

Is the walking rate data proportional? How do you know?

Yes, for every second that passes they walk 1.5 meters.

Will all time / distance relationships always be proportional?

Only if the rate traveled is kept constant; in reality, it would be almost impossible for a person to walk/run/drive/bike/fly for a relatively long distance at a constant rate without slowing down or speeding up.

If a walking rate is 2 meters per second, how does the table of values differ from Yvonne's table of data?

Sample answer: The rate would be 7,200 meters per hour, and the table of values would increase by 7,200 every hour.

If a walking rate is 2 meters per second, how does the graph compare to Yvonne and Lisa's graph? What would be the same? What would be different?

The graph is still proportional, but would have a steeper rate of change (slope).

## QUESTIONS FOR STUDENTS ON TASK (CONTINUED)

If a walking rate were 2 meters/second, what distance can be walked in 40 seconds?

80 meters

How far would Yvonne and Lisa have traveled in the same amount of time?

60 meters

## QUESTIONS TO EXTEND STUDENT THINKING

What would the graph for Yvonne and Lisa's walking rate look like?

*It would start on (0,0) and be a linear relationship.*

If a walking rate was 1 meter/1.5 seconds, how could you find out how fast a person was walking per second?

*Set up a proportional relationship between 1 meter/1.5 seconds and  $\frac{x}{1}$  second. The rate is  $\frac{2}{3}$  meter/second or 0.67 meter/second.*

How would increasing your walking rate affect the values in the table? The graph? The equation?

*In the table you will see the constant rate of change increase. In the graph the slope or steepness of the graph will increase. In the equation the value of  $r$  will increase.*

How would a slower walking rate affect the values in the table? the graph? the equation?

*A table showing a slower walking rate would increase less each hour, the graph would not be as steep, and the value of  $r$  would be less.*

## LESSON SUMMARY

Begin by looking at some of the sketches students created for Question 1. Compare the graphs and what affected their appearances. Discuss that graphs should all show proportional relationships but some graphs will be steeper than others, depending on the walking rates and scales used when graphing. Students may have several approaches to answering the questions. Some students use one rate to solve the problems; others will look at relationships that grow. For example in Question 2, some students use the ratio 3,600 seconds/1 hour; others will use 60 seconds/1 minute and then 60 minutes/1 hour. Have students show how these rates are equivalent. Discuss the equivalency of the rates. Ask students to share the rates they used and compare them to other rates used.

Ask a student to share the equation found for Question 5. As a class, decide on the variables used and what the different parts of the equation represent. Students should be able to identify where the rate appears in the equation. Have some students share how they used their equations to solve Question 6.

## CHECK FOR SUCCESS

- Have students describe the effect on the graph if Yvonne decided to jog to her friend's house. Then students can tell how jogging changes the table and an equation.
- To reinforce their learning, have students explain patterns of change and where they are found in different representations.
- Have students write a description of how they found their own walking rates. Students should also be able to represent their rate in a table and a graph. Students need to be able to explain the pattern of change represented in the table, graph, and equation and state if that change is proportional.
- To verify that they understand the concept, have students give another example of a situation that involves another proportional relationship. Students should name the variables involved, describe what a table of values for the relationship will look like, and explain how the graph will look. Students should then write an equation for the relationship.



# LESSON 1

## WALKING TRIP

In the evenings Yvonne and Lisa enjoy taking walks. Lisa says they walk at a constant rate of 1.5 meters per second. One day they decide they want to walk to a friend's house 10 miles away. Yvonne remembers that 800 meters is about one-half mile. She says it will take them all day to walk. Answer the following questions to decide if the walk to the friend's home will take all day.



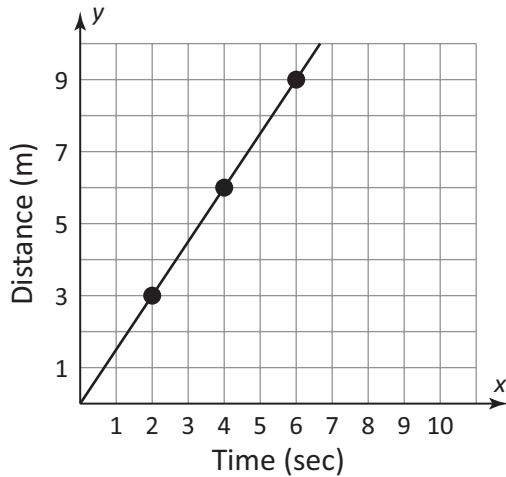
1. Sketch a graph showing Yvonne and Lisa's walking rate.
  
2. How far will Yvonne and Lisa have walked in 1 hour?
  
3. Make a table showing a distance walked, in meters, in 6 hours. What kind of relationship is shown in the table? What does the graph look like?
  
4. How long will it take Yvonne and Lisa to walk 2 miles?

Student \_\_\_\_\_ Class \_\_\_\_\_ Date \_\_\_\_\_

5. How long will it take Yvonne and Lisa to walk 10 miles? Write an equation for how long it will take Yvonne and Lisa to walk any number of miles.
  
6. Using your equation from Question 5, how long will it take Yvonne and Lisa to walk 20 miles? 4 miles?
  
7. Find the rate at which you walk.
  
8. Define the variables you used to determine your walking rate assuming you can continue to walk at this constant rate. Describe the relationships between the variables.
  
9. If you and Yvonne both left at the same time to walk 2 miles to a friend's house, who would get there first?

# STUDENT RECORDING PAGE SOLUTIONS

## 1. Yvonne & Lisa's Walking Rate



## 2. Example 1:

$$\frac{1.5 \text{ m}}{1 \text{ sec}} = \frac{\text{m}}{60 \text{ sec}}$$

$$\frac{1.5 \text{ m}}{1 \text{ sec}} \times \frac{60}{60} = \frac{90 \text{ m}}{60 \text{ sec}}$$

$$\frac{90 \text{ m}}{1 \text{ min}} \times \frac{60}{60} = \frac{5,400 \text{ m}}{60 \text{ min}}$$

## Example 2:

$$\frac{1.5 \text{ m}}{1 \text{ sec}} = \frac{90 \text{ m}}{60 \text{ sec}}$$

$y = 90x$ , where  $y$  represents min  
and  $x$  represents distance

$$y = 90(60)$$

$$y = 5,400$$

5,400 meters per 60 minutes

## 3.

Hours	Meters Walked
1	5,400
2	10,800
3	16,200
4	21,600
5	27,000
6	32,400

Students should notice that the relationship is proportional. The graph shows a linear relationship that passes through the origin.

## 4. $d$ = distance walked = 2 miles; $r$ = walking rate; $t$ = time

If  $\frac{1}{2}$  mile  $\approx$  800 meters then 1 mile  $\approx$  1,600 meters and 2 miles  $\approx$  3,200 meters.

If in 60 minutes, the distance is 5,400 meters, then  $5,400 \div 60$  gives the rate for 1 minute. Rate for 1 minute is 90 meters/min. Now use  $d = rt$ .

STUDENT RECORDING PAGE SOLUTIONS (CONTINUED)

4. (continued)

$$3,200 = 90r$$

$35.6 \approx r$  It will take about  $35\frac{1}{2}$  minutes to walk 2 miles.

Another method students can use to answer this question:

$$\frac{1,600 \text{ m}}{1 \text{ mi}} = \frac{?}{2 \text{ mi}} \quad \frac{1,600 \text{ m}}{1 \text{ mi}} \times \frac{2}{2} = \frac{3,200 \text{ m}}{2 \text{ mi}}$$

From the table, I see that they walk a rate of 5,400m/1 h. I set up a proportion.

$$\frac{5,400}{1 \text{ h}} = \frac{3,200}{?}$$

$$3,200 \div 5,400 \approx 0.593$$

$$\frac{5,400 \text{ m}}{60 \text{ min}} \times \frac{0.593}{0.593} = \frac{3,200 \text{ m}}{35.5 \text{ min}}$$

5. Example 1:

$$\frac{2 \text{ mi}}{35\frac{1}{2} \text{ min}} = \frac{10 \text{ mi}}{X}$$

$$\frac{2 \text{ mi}}{35\frac{1}{2} \text{ min}} \times \frac{5}{5} = \frac{10 \text{ mi}}{177\frac{1}{2} \text{ min}}$$

$$\frac{60 \text{ min}}{1 \text{ hr}} \quad 177.5 \div 60 = 2.96 \text{ hrs}$$

It will take them about 3 h to walk 10 miles.

$$\frac{2 \text{ mi}}{35\frac{1}{2} \text{ min}} = \frac{1 \text{ m}}{17.75 \text{ min}} \quad \text{So, } y = 17.75x$$

Example 2:

$x$ (min)	$y$ (min)
1	17.75
2	35.5
3	53.25
4	71
5	88.75
6	106.5
$x$	$17.75(x)$

$$y = 17.75x$$

$$y = 17.75(10) = 177.5$$

It will take 177.5 min or 2 h 57.5 min

6. When  $x = 20$  miles, it will take about 355 minutes or 5 hours 55 minutes

When  $x = 4$  miles, it will take about 71 minutes or 1 hour 11 minutes

7. Answers will vary. Walking rates should be in the 2 meters/second range.

8. The variables are time and distance. The relationship is proportional. As the time increases by 1 hour, the distance they walk increases 5,400 meters. The relationship is also linear. If I graph the relationship, I get points that lie on a straight line.

9. Answers will vary; I know it takes Yvonne  $35\frac{1}{2}$  minutes to walk 2 miles.