Reproducibles and Additional Reference Tools

- 3.1 Developing Theories: Template
- 4.1 CRA Assessment: Template
- 4.2 CRA Assessment Example: First Grade
- 4.3 CRA Assessment Example: Third Grade
- 4.4 CRA Assessment Example: Fifth Grade
- 4.5 CRA Assessment Example: Seventh Grade
- 5.1 Collaborative Study: Template
- 6.1 Planning a Student Interview: Template
- 6.2 Conducting a Student Interview: Template
- 6.3 Analyzing a Student Interview: Template
- 6.4 Parent Consent Form
- 8.1 Main Lesson–Menu Lesson Plan: Template
- 8.2 Main Lesson–Menu Lesson Plan Example: First Grade
- 8.3 Main Lesson–Menu Lesson Plan Example: Third Grade
- 8.4 Main Lesson–Menu Lesson Plan Example: Fifth Grade
- 8.5 Main Lesson–Menu Lesson Plan Example: Seventh Grade

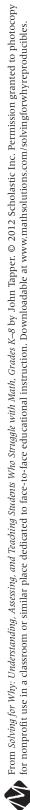
All Reproducibles are also available as downloadable, printable versions at www.mathsolutions.com/solvingforwhyreproducibles.

Developing Theories: Template

True de la come de cost	
I want to know more about[student's name]	math thinking, because
he/she	
[Specific performance/behavior that ca	aused concern]
on[assignment or assessment	 .t]
1. Theory from the Learner Frame	
What conceptual tools (models) does the student use (if any) to un	iderstand this concept?
What evidence exists (if any) that a learner struggles with memor	y, attention, or anxiety challenges?
2. Theory from the Content Frame	
Which gaps in math understanding might account for this learne he or she need to understand mathematically not to struggle with	
3. Theory from the Instructional Frame	
How might this learner have been taught this concept? In what w have led to misunderstandings? What other approach might bette	
What the student might think is	
1	,
because	
2	,
because	
3	
because	

CRA Assessment: Template

CRA Assessment Template		
BIG IDEA/FOCUS		
PLANNING		
Standards		
What standards will the assessment address?		
Expectations		
What do you think students already know about this topic?		
What kinds of models would you expect students to use?		
Where might they have difficulty?		



(continued)

ADMINISTERING THE ASSESSMENT

How will you administer the assessment? (Student choice or teacher choice for starting station? Moving individually? Rotating groups? Whole class?)

Concrete Station	
Problem	
Materials	
How will you record	
student work?	
Representational Stati	ion
Problem	
Materials	
How will you record	
student work?	
Abstract Station	
Problem	
Materials	
How will you record	
student work?	

Sorting categories	
When sorting the student worl	k by models, strategies, or algorithms, what patterns did you find?
Models	
Strategies	
Algorithms	
Questions Pile	
What questions did you	
have about samples that were difficult to	
understand?	
What (if any) common nisconceptions appeared	
in the work?	
Did students show any	
indicators of cognitive difficulties, such as	
nemory difficulties,	
attention challenges, or anxiety? If so, who and	
what were the indicators?	
Anything else discovered with this assessment?	

CRA Assessment Example: First Grade

CRA Assessment Template

BIG IDEA/FOCUS

regrouping with addition to 30

PLANNING	
Standards	
What standards will the assessment address? Common Core, Grade 1	(1) Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations.
Expectations	
What do you think students already know about this topic?	We've practiced trading tens for 10 ones. They also (most of them) know how to count to 30.
What kinds of models would you expect students to use?	We've been using interlocking cubes and base ten blocks, so I expect them to use these.
Where might they have difficulty?	I think about half the class will have no trouble solving the problem, but I expect them to count on, rather than regroup.

ADMINISTERING THE ASSESSMENT

How will you administer the assessment? (Student choice or teacher choice for where to start? Moving individually? Rotating groups? Whole class?)

I've put them into three groups (mixed ability). They'll move from one station to the next every ten minutes. I'll stay at the concrete station and the learning specialist will be at the abstract station.

Concrete Station	
Problem	Sam has 12 books. Edie has 9 books. How many books do they have together? Show me, then draw a picture.
Materials	interlocking cubes, base ten blocks, pennies and dimes, tiles, and books
How will you record student work?	I'll ask the children to draw their solutions. I'll ask the students to draw their solutions and explain their answers in their own words.
Representational St	ation
Problem	Jason has 7 cents. He gets 14 more cents. How much money does he have now? Draw a picture and use numbers.
Materials	pencils, erasers, paper (plain, lined, graph), markers
How will you record student work?	I'll ask the children to draw their solutions, once they have built them with one of the manipulatives. I'll also have a clipboard and 5-by-7-inch sticky notes on which I'll make notes.
Abstract Station	
Problem	Alissa has 18 stickers. Her mom gives her 7 more. How many does she have now? Write a number sentence to answer.
Materials	pencils, paper, the chart where we wrote our daily equations during meeting (to remind students what a number sentence is)
How will you record student work?	Students will write their number sentence and add their names.

ANALYZING THE ASSESSMENT Sorting categories When sorting the student work by models, strategies, or algorithms, what patterns did you find? Models The most common model was tally marks at the representational station (twelve students). I think this was because we just worked on tallies in our unit on counting. The rest of the students (who finished) drew some kind of circle. Many (eight) children used books at the concrete station. This isn't surprising since the problem was about books. Strategies Students used counting all frequently at both C and R stations. A few students counted on. I marked on the papers when I noticed this. (CA = counting all, CO = counting on.) Only three students made groups of ten at either the concrete or representational stations. Even when they used groups of five for tallies they still counted by fives, rather than putting two together to make ten. Algorithms Four students (Gina, Amy, Joel, Milo) wrote a number sentence to solve. It's unclear from their work whether they are regrouping. Kay (learning specialist) says that they were all counting on from 18 except for Milo, who started at 1. **Questions Pile** What questions did you Abby: I want to check in with her on her counting. She showed she could use have about samples both cubes and drawing (drew circles for pennies) but her counts were wrong that were difficult to in both cases. Is there a counting problem or is she rushing? Inattentive? understand? Ben: At the concrete station he piled up books. He did make two piles but he didn't seem to connect this to the numbers in the problem. He was unable to complete the other problems. At the representational station he drew what I think is a dollar bill. What (if any) common All of the students except Abby and Ben seemed to have the sense of misconceptions appeared "having some" and "getting more." Abby seemed to have difficulty in the work? counting and Ben just seemed stuck. (See above) Did students show any Ben might have been anxious or distracted. He looked a little sad indicators of cognitive and lost and didn't do much with these problems, even when difficulties, such as I offered to scribe for him. memory difficulties, attention challenges, or anxiety? If so, who and what were the indicators? Anything else discovered Next time I give it, I should remember to have an activity for students with this assessment? who finish early. Several students were sitting around waiting for us to change stations. Waiting is not a good use of their time.

CRA Assessment Example: Third Grade

CRA Assessment Template

BIG IDEA/FOCUS

Can students use a multiplicative model to solve division problems with remainders?

PLANNING	
Standards	
What standards will the assessment address? Common Core, Grade 3	3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement guantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
Expectations What do you think	I Know that students have used circles and stars, beans and cups, and area as
students already know about this topic?	models for multiplication. What I don't know is how they can apply this to sharing among groups.
What kinds of models would you expect students to use?	I think they will use circles and stars most often, though several students seem to get the connection between that model and the area model. I would expect these students to draw rectangles because this is faster (and easier) than drawing lots of stars.
Where might they have difficulty?	I think some students might get stuck on the remainder piece. I expect most of them to have little trouble putting a total into groups.

ADMINISTERING THE ASSESSMENT

How will you administer the assessment? (Student choice or teacher choice for where to start? Moving individually? Rotating groups? Whole class?)

I'm going to set up three stations and have students move from station to station as they finish. I'll let them choose where they start this time.

I'll have trays at each station for finished student work. I'll stay at the concrete station and Stella (classroom para) will stay at the representational station.

Concrete Station	
Problem	Our class is going on a field trip. There are 24 students in our class. If each car can hold 5 students, how many cars will we need to go on our trip? Show your thinking with one of the materials.
Materials	cubes, tiles, beans, chips
How will you record student work?	I'm going to take a picture of each student's work, print it limmediately) and have the student write about it for me. I'll have them do the writing at a second table.
Representational St	ation
Problem	There are 48 children in third grade. All of third grade is going on a field trip. If mini-buses hold 10 kids each, how many buses will we need? Draw a picture to show your thinking.
Materials	colored pencils, markers, paper, graph paper
How will you record student work?	Students will use representations and record their solutions. Stella will ask guestions and take notes as necessary.
Abstract Station	
Problem	There are 39 children in second grade. They are all going on a field trip. If parent cars hold 6 children each, how many cars are needed for the field trip? Show your thinking only with numbers. (Write an equation or number sentence.)
Materials	Pencils and paper
How will you record student work?	Students will record equations to show their solutions.

Sorting categories	
When sorting the student work by models, strategies, or algorithms, what patterns did you find?	
Models	Students seemed to have the easiest time at the representational station. Ten kids drew some kind of circle with something like tallies in it (not stars). No one used a rectangle. At the concrete station students used piles of beans (eleven) or stacks of cubes (eight).
	One student, Sam, didn't seem to understand that the groups all needed to be equal-except for the remainder. He put different numbers of kids into cars (concrete) but explained it by saying that cars have different numbers of seats.
Strategies	Mostly students used counting out individual groups until reaching a total. A few students (two) started with a guess at the number of groups and then tried giving tallies to them one at a time.
Algorithms	This was a surprise. Only one student (Evan) used multiplication. All others who had answers used repeated addition.
Questions Pile	
What questions did you have about samples that were difficult to understand?	Jessica's drawing (representations) doesn't make sense to me. The boxes she made look like cubes but there are too few and she doesn't indicate an answer. Did she stop? Give up? Taylor's abstract page just has numbers on it (6 and 39). I don't know if he intended to do something with them or if this is as far as he got.
What (if any) common misconceptions appeared in the work?	The remainders were a problem for most students (twelve). This isn't so much a misconceptio as something that we haven't done yet.
Did students show any indicators of cognitive difficulties, such as memory difficulties, attention challenges, or anxiety? If so, who and what were the indicators?	Erika seemed frustrated when she couldn't think of an equation. She asked several times, "What's an equation?" When we gave her examples she still seemed puzzled. I don't think she was confused about equations, I think she was frustrated because she couldn't come up with any.
Anything else discovered with this assessment?	I expected the kids to make connections with the work we've done on grouping. While they used the groups that the problem asked for, most just counted on. They don't seem to hav a sense of how to use the models for problems yet.

CRA Assessment Example: Fifth Grade

CRA Assessment Template

BIG IDEA/FOCUS

finding quotients with double-digit divisors

PLANNING	
Standards	
What standards will the assessment address? Common Core, Grade 5	6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Expectations	
What do you think students already know about this topic?	Students have already demonstrated the ability to compute quotients with single digit divisors using a variety of strategies as well as both standard and partial quotients algorithms.
What kinds of models would you expect students to use?	We've done quite a lot of work with the area model and the area strategy for decomposing numbers during multiplication. I hope they will apply this to division. Also, the fundamental idea that division is about making equal groups should help students. In a sense this is a test of their multiplicative reasoning.
Where might they have difficulty?	There are a couple of students who only use algorithms. I wonder if they will be able to create models for the problems. They might know how algorithms are informed by number sense but this will test their understanding.

ADMINISTERING THE ASSESSMENT

How will you administer the assessment? (Student choice or teacher choice for where to start? Moving individually? Rotating groups? Whole class?)

The assessment will be conducted as a whole class. We'll start with the concrete models and work our way to the abstract. Students will work at their tables in order to share materials but they must work independently. I think this can happen in one class period.

Concrete Station	
Problem	Suzanne has \$5.00. She wants to buy cupcakes that cost \$.36 each. How many cupcakes can she buy?
Materials	classroom money, base ten blocks, chips for trading
How will you record student work?	Students must first create a model using materials and then draw a picture to show how they solved.
Representational St	ation
Problem	There are 14 candies in every roll. How many rolls can you make with 800 candies?
Materials	graph paper, unlined paper, pencils, markers
How will you record student work?	Students will record their representations and their explanations. I will remind students of the representations we've worked with already (graphs, T-charts, tables, organized lists, diagrams) with a chart of these at the front of the room that includes examples. I think this will help students whose first language is not English with the idea of "representations."
Abstract Station	
Problem	196 students signed up for soccer. If the soccer league puts 16 students on a team, how many teams will the league have?
Materials	pencils and paper
How will you record student work?	Students will record an equation <u>and their calculations</u> so I get a sense of how they computed their answers.

ANALYZING THE ASSESSMENT

Sorting categories

When sorting the student work by models, strategies, or algorithms, what patterns did you find?

when sorting the student w	ork by models, strategies, or algorithms, what patterns did you find?
Models	Not a surprise Everyone went for the money. I thought this would happen but I wasn't prepared for how many didn't really know how much the coins were worth or how to add coins up to a dollar. There was a good deal of whispering at tables as students reminded each other of how to make 35 cents. Two students began by using pennies (!).
	A T-chart was the most common representation. Students put candies one side and rolls on the other.
Strategies	Except at the abstract station, virtually everyone used some form of "adding up." They would start with one cupcake or roll of candies and then work from there. About half the class figured out that they could save time by using multiples.
Algorithms	Nine of the twenty-five students in the class used some form of algorithm to solve the problem. The rest really used another counting up strategy, similar to what they did at the othe stations. They don't seem to have generalized the idea of the algorithm to larger numbers.
Questions Pile	·
What questions did you have about samples that were difficult to understand?	Anog seemed confused at all parts of the assessment. This is not surprising since his English is still fairly rudimentary. During the abstract part of the assessment he wrote an equation (I think) that might be the Thai version of division. This needs more investigation with the teacher supporting him in learning English.
	Samantha and Tyler had difficulty with the money. The drawings on their papers looked like the others at their table but when I watched them they seemed lost. I should check on their understanding of money.
What (if any) common	The value of coins was not really a misconception—just something a few students don't know.
misconceptions appeared in the work?	I was surprised at how the students continued to "add up" rather than use multiplication. Why?
Did students show any indicators of cognitive difficulties, such as memory difficulties, attention challenges, or anxiety? If so, who and what were the indicators?	Anog seemed anxious, but it's hard to know how much of that is simply that he is in a new country.
	Gerald was fidgety throughout all parts of the assessment. He was bouncing his foot and waving his pencil constantly. It didn't seem to bother his tablemates but I'm concerned it was distracting.
	The difficulty with money makes me wonder whether these students were taught about money and might have forgotten? Could this be a memory issue or something I should speak with the fourth-grade teacher about?
Anything else discovered with this assessment?	Using a money problem for the concrete portion was useful in the sense that we had plenty of manipulatives and it revealed a weakness some students have there. I wonder, though, as students were working on the problem if it might not really fit the focus of the assessment since it added the aspect of decimals.

CRA Assessment Example: Seventh Grade

CRA Assessment Template

BIG IDEA/FOCUS

solving a distance/rate problem with an initial position-moving from d = r*t (a proportion, to d = r*t + s (starting position)

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers.
Solve equations of these forms fluently.
Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?
Students have spent a considerable amount of time over the last two years working on a variety of models for proportions. They have not written these in the <i>y</i> = <i>kx</i> form, however. Earlier in the year we wrote proportions as an equivalence of ratios. They have some experience with rate problems from sixth grade.
I'm not sure what students really understand about equivalence.
I don't think physical materials would be useful for this CRA. I'm substituting a SimCalc simulation of distance/time graphing for a concrete manipulative. While virtual, it still allows students to use a graphing simulation to find an answer.
I don't know if the problems will be so difficult, but I'm not sure if the students will see the connections with the equation for a linear function. I also think they might have some trouble separating the starting position (constant) from the rate.

ADMINISTERING THE ASSESSMENT

How will you administer the assessment? (Student choice or teacher choice for where to start? Moving individually? Rotating groups? Whole class?)

Students will work at individual stations for each of the problems. They can choose where to start. The four computers that have SimCalc on them are connected to printers so students can print out screenshots. There may be a little backlog at the computers but all the students will eventually work their way through.

Concrete Station	
Problem	Use the SimCalc simulation on the classroom computers to show a solution to the problem below.
	An elevator moves upward at a steady rate of 1 floor every 3 seconds. If the elevator starts on the third floor, how long will it take to reach the 11th floor?
	Print the screen shot of your solution and write about it.
Materials	computers, SimCalc software (SimCalc MathWords® can be retrieved at: www.kaputcenter.umassd.edu/products/software/)
How will you record student work?	printed screenshot and explanation
Representational St	ation
Problem	Tom and Mary are having a race. Mary runs 1 meter every 1.2 seconds. Tom runs 1 meter every 1.5 seconds. If Mary gives Tom a 4 meter head start, who will win the race? Show your work with a representation (chart, graph, or table).
Materials	graph paper, paper, colored pencils, pencils
How will you record student work?	Students will submit their representations and explanations along with their solutions.
Abstract Station	
Problem	Lola runs the 100-yard dash. When she runs the whole distance her time is usually 12 seconds. If she starts at the 40-yard line (gets a 40-yard head start) how long will it take her to run 100 yards?
Materials	pencils and paper
How will you record student work?	Students will record answers (figured with equations) and explanations for their work. All computations will be included.

ANALYZING THE ASSESSMENT

Sorting categories

When sorting the student work by models, strategies, or algorithms, what patterns did you find?

when sorting the student w	ork by moaels, strategies, or algorithms, what patterns all you fina?
Models	The elevator simulation produced a variety of interesting results. All the students (but one) were able to create the simulation adequately. They all were not, though, able to interpret the graph for the elevator's motion. There were several comments on the screenshots about the fact that the graph was a "slanted line" (rather than a vertical line like the motion of an elevator). Some students seemed also to be confused with the difference between the 11th floor and <i>going up</i> 11 floors. The representational problem was meant to elicit mathematical representations but several students attempted to solve the problem using a drawing of the situation and trying to reason their way through it. Most were unsuccessful. It would seem that this problem needs attention to help students visualize what's actually taking place.
Strategies	The most common strategies came up at the representational station and the abstract station. Students made charts that showed Mary and Tom's position ("How far," "distance," "distance gone") with relative times. Four students also made a graph of this. I think this represents sophisticated thinking to see the table as a graph with Tom and Mary's progress as separate lines. I wonder if the SimCalc program helped with this. (Did these students do that first?) When working at the abstract station, students solved the equation in two parts.
Algorithms	No one created a <i>d=r*t + s</i> equation. Many students put drawings in at the abstract station, even though they were supposed to use only equations. For students who solved the problem, the approach was to use one equation to find Lola's rate and then to apply it to 60 yards instead of 100.
Questions Pile	
What questions did you have about samples that were difficult to understand?	 I have a number of questions to follow up on: Zach's screen shot from the SimCalc station shows the elevator at the 11th floor and nothing else. I have no idea how he made it do that or what sense he made of it since all he wrote was, "It's on the 11 floor." Chris drew an oval for the race problem with a big "12" in the center but nothing else. Is the oval a racetrack? Did he just not know how to go further? Junie was able to print an accurate screenshot of the elevator problem. He circled the 11th floor and wrote, "24." He was unable to do very much with the other problems. Is this an issue with English?
What (if any) common misconceptions appeared in the work?	The most common difficulty students had was being thrown off by the constant. I think this element made the problems unrecognizable as proportional problems for almost everyone.
Did students show any indicators of cognitive difficulties, such as memory difficulties, attention challenges, or anxiety? If so, who and what were the indicators?	Jocelyne didn't get finished at any of the stations. This is fairly typical of her work. I'm not sure if there is an issue but it deserves some investigation. Junie seemed to have some difficulty with the language of the problems. His English in class is pretty good so I often forget that he is still learning English. I should have read the problems to him or had him work with Jaime (so they could speak about the contexts in Spanish) to be sure he understood what he was supposed to do.
Anything else discovered with this assessment?	Next time I give it, I should remember to have an activity for students who finish early. Several students were sitting around waiting for us to change stations. Waiting is not a good use of their time.

REPRODUCIBLE 5.1

Collaborative Study: Template

Student's name:	Student's grade:
Student's teacher:	
Date of referral for collaborative study:	
Who made the referral?	
Has proper consent ¹ been given to gather and share this information?)
yesno	
By whom?	Date
What behavior/performance prompted a desire for collaborative stud	y?
On which particular math content is this study focused?	

I. Student History

• Impressions from the math teacher

Report important comments from the student's math teacher about both performance/behavior on the targeted math area and overall impressions in class. Please collect samples of student work to share at the collaborative study meeting.

• Math report card assessment (and/or grades) over time

Report student's report card assessments over the last three years. Note any patterns that exist.

• Standardized Test Scores

Years		
Results		

¹The required consent will vary depending on student's IEP or 504 status, the information gathered, and state and local policies and statutes. Please consult with school leaders to be sure what constitutes proper consent.

• Other relevant information on the student's math history?

II. Setting Up the Collaborative Study

• Who will attend?

(Suggestions: student's math teacher, school leader, math specialist, another math teacher)

- Who will notify attendees?
- Who will facilitate the meeting?
- When and where will the meeting take place?

Date: ______
Time: ______
Location: _____

III. Meeting and Intervention

- 1. Identify a specific concern about the student.
- 2. Give background on the student (allow members to ask clarifying questions).
- 3. Share recent student work.
- 4. Brainstorm theories that might account for the student's struggles.
- 5. Create an action plan to test the group's theories.
- What theories does the group have about the student's difficulties? (*Be sure to connect the theories to evidence.*)

• What interventions might be used to test these theories?

Theory 1:

Theory 2:

Theory 3:

• Who will implement these interventions and gather data/impressions on their effectiveness?

Theory 1:

Theory 2:

Theory 3:

• Follow-up meeting to review results:

Theory 1:

Theory 2:

Theory 3:

IV. Results and Recommendations

- Report results of intervention:
- Recommendations of the collaborative study committee:

Planning a Student Interview: Template

Student's name:	Student's grade:
Student's teacher:	
Has proper consent ¹ been given to gather and share this information?	2
yes no	
By whom?	Date
On which particular math content is this interview focused?	

I. Student History

• Impressions from the math teacher

Report important comments from the student's math teacher about both performance/behavior on the targeted math area and overall impressions in class. Please collect samples of student work to share at the collaborative study meeting.

• Math report card assessment (and/or grades) over time

Report student's report card assessments over the last three years. Note any patterns that exist.

• Standardized Test Scores

Years		
Results		

• Conversation with a parent or guardian

Report impressions/information that the student's parent or guardian has about the student's math learning.

¹The required consent will vary depending on student's IEP or 504 status, the information gathered, and state and local policies and statutes. Please consult with school leaders to be sure what constitutes proper consent.

• Other relevant information on the student's math history?

II. Problems to Use for the Interview

- Math focus for problems for the interview:
- Approximate grade level of student on this topic (your estimation):
- Problems at grade level: *Problem 1:*

Problem 2:

Problem 3:

• Problems one grade below grade level: *Problem 1:*

Problem 2:

• Problems two grades below grade level: *Problem 1:*

Problem 2:

• Problems one grade above grade level: *Problem 1:*

Problem 2:

III. Setting Up the Student Interview

Make sure that the classroom teacher has advanced notification. Try to avoid removing the student from favorite activities, like recess. Is the interview space appropriate? Are you unlikely to be interrupted?

Time:	

Location: _____

Conducting a Student Interview: Template

Student's name: _____

Student's grade: _____

Student's teacher: _____

Remember that the goal is to understand student thinking, not to teach. Recording time will help you find places of interest in the interview more easily.

Begin Recording

Recording Time: 00:00

• Introductions

Create an informal and supportive atmosphere with a conversation about the student's interests. Be sure to get the student's impressions of why the interview is taking place and let him/her know what he/she might expect. Emphasize that you are more interested in thinking than in correct answers. For this reason it will be helpful to you if the student can think aloud while working.

Recording Time:

• Problem 1:

Read the problem <u>together</u>. Begin by asking the student to tell you what the problem is asking you to find out. As the student works and thinks aloud, record impressions below as well as follow up questions. The **>** *prompt is a space for you to record questions you want to remember to ask the student.*

► Remember to ask:

Impressions:

Save student sample for Problem 1.



Recording Time:

• Problem 2:

Read the problem <u>together</u>. Begin by asking the student to tell you what the problem is asking you to find out. As the student works and thinks-aloud, record impressions below as well as follow up questions. The **>** *prompt is a space for you to record questions you want to remember to ask the student.*

► Remember to ask:

Impressions:

Save student sample for Problem 2.

Recording Time:

• Problem 3:

Read the problem <u>together</u>. Begin by asking the student to tell you what the problem is asking you to find out. As the student works and thinks-aloud, record impressions below as well as follow up questions. The **>** *prompt is a space for you to record questions you want to remember to ask the student.*

► Remember to ask:

Impressions:

Save student sample for Problem 3.

Recording Time:

• Problem 4:

Read the problem <u>together</u>. Begin by asking the student to tell you what the problem is asking you to find out. As the student works and thinks-aloud, record impressions below as well as follow up questions. The **>** *prompt is a space for you to record questions you want to remember to ask the student.*

► Remember to ask:

Impressions:

Save student sample for Problem 4.

Continue template as needed for subsequent problems.

Analyzing a Student Interview: Template

Student's name: _____

Student's grade: _____

Student's teacher: _____

I. Prepare for Analysis

Gather the following:

- Recording of interview
- Interview template recording sheets (see Reproducible 6.2)
- Student history (see Reproducible 6.1)

II. Listen to Interview Recording

- Listen to the interview, making frequent stops to add more insights to the "Impressions" section of the Student Interview template for each problem.
- Supplement your on-the-spot impressions with connections to the Student History.

III. Develop Theories

Below articulate theories that might explain student difficulty with the target math concept. Using the learner, content, and instruction frames (see Chapter 3) may help. Evidence should link directly to <u>quotes from the interview</u> (the time on the recording is helpful), <u>impressions you recorded</u>, <u>student history</u>, or <u>examples from the student work</u>.

Theory 1:

Evidence:

Intervention that could test this theory:

Theory 2:

Evidence:

Intervention that could test this theory:

Theory 3:

Evidence:

Intervention that could test this theory:

Parent Consent Form

Dear (Parent's or Guardian's name),

I have noticed that your child, ______, has had some difficulty with math this year. Since our school wants every child to be successful, we would like to help your child by finding out more about his/her math thinking. To do this, we would like to have ______ (interviewer's name) interview ______ (child's name) to learn more about the way

(s)he understands math.

I would be happy to meet with you to explain the interview process and show you what _______ (child's name) can expect during the interview. If there is any part of the process that makes you uncomfortable, I am happy to talk about it with you. We encourage you to ask questions if you have any. Your child's participation in this interview is voluntary; however, we believe he/she will enjoy the process and benefit from it. Our hope is that the interview with your child will give us information that we can use to teach him/her more successfully.

The interview will take place on ______, at _____. It will last for about an hour. We welcome the opportunity to meet with you after the interview to discuss what we have found and to get your take on our findings.

If you have any questions about this process, please do not hesitate to contact me.

_____ (interviewer's name)

_____ (school phone number)

_____ (email address)

I ______ (parent's name) give permission for my child to be interviewed by ______ (interviewer's name) for the purpose of understanding his/her mathematical thinking. I understand that this interview may be audio/videotaped for the purpose of review. I understand that recordings of the interview will be used by (here list everyone who may be involved in analysis). Any recordings of this interview will be destroyed by the end of the year.

_____ (parent's name)

_____ (date)

Main Lesson—Menu Lesson Plan: Template

Main Lesson—Menu Lesson Plan				
BIG IDEA/FOCUS				
Launch (5–10 minutes)	Inquiry or problem for the launch:	Ask:		
Think:	5			
How will this activate prior knowledge <u>and engage every</u> <u>student</u> ?				
How will it lead into the main lesson?				
Explore/Main Lesson (about 20 minutes)	Inquiry or problem for the main lesson:	Ask:		
Think:				
How can I provide opportunities for students to talk to each other about their math thinking during this lesson? How will I support all students to be part of this experience?				
Are the students doing the math work—or am I doing it for them?				

Menu (about 30 minutes)

Think:

How will I differentiate this menu work for the variety of learners in my class?

How will I collect and respond to this work?

• Follow up from main lesson:	• Number and operations work:	• Problem to solve:	
	Note: Students choose one or the other. One dot is easier and two dots is more challenging.		
		Note: Students choose one or the other. One dot is easier and two dots is more challenging.	
 Game to reinforce or investigate the big idea: 	• Math Journal:		

Closure (about 10 minutes)	Ask:	
Think:		
What question can I ask, or problem can I pose, to focus attention and discussion on the work from the last hour?		
What do I want students to remember about this lesson?		
<i>How will I record student thinking?</i>		

Main Lesson—Menu Lesson Plan Example: First Grade

Main Lesson—Menu Lesson Plan BIG IDEA/FOCUS comparing two or more numbers less than 20		
Explore/Main Lesson (about 20 minutes) Think: How can I provide opportunities for students to talk to each other about their math thinking during this lesson? How will I support all students to be part of this experience? Are the students doing the math work—or am I doing it for them?	Inquiry or problem for the main lesson: Put students into groups of 4 (make sure there's a mixed group for the math coach to work with today) Each group gets 17 red cubes, 12 blue cubes, and 9 orange cubes. Don't stack beforehand.	Ask: 1. Which has more, red or blue? How many more? How do you know? (small group, whole group: record on chart paper) 2. Which has more, orange or blue? How many more? How do you know? (same) 3. Put these in order. How do you know it's the right order? (Have groups show their results)

Menu (about 30 minutes)

Think:

How will I differentiate this menu work for the variety of learners in my class?

How will I collect and respond to this work?

• Follow up from main lesson:	• Number and operations work:	• Problem to solve:
 Optional (for those who want a challenge) How many <u>more</u> cubes (all together) would you need so that everyone in our cubes problem would have the same amount of cubes? 	• $17 - 12 =$ •• $17 - 9 =$ $17 - 9 =$ $27 - 9 =$ $12 - 9 =$ $37 - 9 =$ $10 + 7 =$ $24 + 26 =$ $12 + 9 =$ $23 + 27 =$ Note: Students choose one or the other. One dot is easier and two dots is more challenging.	One dot: Tim has 8 pennies. Sam has 11 pennies. Who has more? How many more? Use numbers, words, and pictures. Two dots: Jose has 28 pennies. Karen has 9 pennies. Who has more? How many more? Use numbers, words, and pictures. Note: Students choose one or the other. One dot is easier and two dots is more challenging.
 Game to reinforce or investigate the big idea: Red and black with no face cards. Players divide the deck among them evenly. Any extra cards are discarded (as are face cards). Aces are "one." Each player puts the top car in their hand in play. Whoever has the highest card wins all the cards played. If there is a tie, players put another card down and the highest card wins all the cards out. Play continues until one player has all the cards. 	• Math Journal: Put a picture of a bar graph in the journals. One bar with 8 squares, one with 5 squares. Prompt below graph: Which has more? How many more? How do you know? Remember to read this for the students who can't do it alone. Transcribe for Edith and Jerome.	

Closure (about 10 minutes)	Ask:	
Think:	How can you tell when one	
What question can I ask, or problem can I pose, to focus	group has more than another? (Write down ideas for this)	
attention and discussion on the work from the last hour?	If possible, highlight any	
What do I want students to	connections with subtraction	
remember about this lesson?	that they make.	
How will I record student	How might counting help you	
thinking?	with knowing if one group has more?	
	Use chart paper to record	
	student thinking.	



Main Lesson—Menu Lesson Plan Example: Third Grade

Main Lesson—Menu Lesson Plan BIG IDEA/FOCUS multiplying numbers greater than 10		
Explore/Main Lesson (about 20 minutes) Think: How can I provide opportunities for students to talk to each other about their math thinking during this lesson? How will I support all students to be part of this experience? Are the students doing the math work—or am I doing it for them?	Inquiry or problem for the main lesson: Start with: A gross is a dozen dozens. How many is that? Follow up with: How many eggs would be in 24 dozens?	Ask: Be sure to mix the groups up so the students can help each other if they get stuck. Small group first, then big group sharing. Make sure to have counters out for the students who want to use them. Maybe egg cartons, too?

Menu (about 30 minutes)

Think:

How will I differentiate this menu work for the variety of learners in my class?

How will I collect and respond to this work?

• Follow up from main lesson:	• Number and operations work:	• Problem to solve:
There are 14 candies in every roll of candies. How many rolls of candies would we have to buy for everyone to get one?	• 6 × 7 = •• 10 × 6 = 10 × 4 = 12 × 5 = 8 × 6 = 7 × 12 = 83 - 17 = 350 - 147 = 100 - 35 = 400 - 189 = Note: Students choose one or the other. One dot is easier and two dots is more challenging.	 Rugs are \$10 a square foot. If I want to buy a rug that is 6 feet by 4 feet, how much will it cost? Rugs are \$117 a square foot. How much would it cost to put a rug in this room? 7 4 9 9 9 Note: Students choose one or the other. One dot is easier and two dots is more challenging.
 Game to reinforce or investigate the big idea: 	• Math Journal:	
Double Dice: Each player rolls her dice at the same time as her opponent. When the dice hit the table, the players multiply their dice together. Whoever has the higher total, wins. Graph paper is used to verify solutions when there is a dispute. For example, if one player rolls a 6 and a 4 and the other player rolls two 5s, the players may have to draw rectangles (something they learned in the main lesson) to determine which roll (6 times 4 or 5 times 5) is higher. Players play until one player has won ten rolls.	How is multiplying by numbers greater than 10 the same or different than multiplying by numbers less than 10? > Make sure to check in with Ben and Ashley to be sure they don't need help with the writing.	

	4 -1-	
Closure (about 10 minutes)	ASK:	
Closure (about 10 minutes) Think: What question can I ask, or problem can I pose, to focus attention and discussion on the work from the last hour? What do I want students to remember about this lesson? How will I record student thinking?	Ask: What did you find out about multiplying by numbers bigger than 10? Try to get them to focus on decomposition. How is multiplying by numbers over 10 "different" than times tables? Record student thinking where everyone can see it.	

Main Lesson—Menu Lesson Plan Example: Fifth Grade

Main Lesson—Menu Lesson Plan BIG IDEA/FOCUS using equivalent fractions to add or subtract fractions with unlike denominators		
How will it lead into the main lesson?	share. Keep a running list on the board.	
Explore/Main Lesson (about 20 minutes)	Inquiry or problem for the main lesson:	Ask:
Think: How can I provide opportunities for students to talk to each other about their math thinking during this lesson? How will I support all students to be part of this experience? Are the students doing the math	Heather and Brenan are eating a pan of brownies. Heather ate ¹ /4 of the brownies. Brenan at ⁵ /8 of the brownies. Did they eat the whole pan of brownies? Students should work in table groups to come up with an answer. (Everyone must be able to explain the group answer.) One member should put their solution (and evidence) up on the board.	Whole group debrief: What strategies did groups use for this problem? How are they the same (different)? What did you notice in another group' approach that was interesting? (If needed, ask each group to come up with something.)
work—or am I doing it for them?	Apply any of the strategies that groups used individually for: Tina said she ate ⁹ /16 of the cake. Al said he ate ² /3 of the cake. Could they both be telling the truth? Did they eat the whole cake? Who ate more? Let students get started on this before starting menu so anyone who is stuck	

at the beginning can be helped.

Menu (about 30 minutes)

Think:

How will I differentiate this menu work for the variety of learners in my class?

How will I collect and respond to this work?

• Follow up from main lesson:	• Number and operations work:	• Problem to solve:
Tina said she ate ⁹ /16 of the cake. Al said he ate ² /3 of the cake. Could they both be telling the truth? Did they eat the whole cake? Who ate more?	• $1/4 + 2/4 =$ •• $1/4 + 1/2 =$ $1/5 + 3/5 =$ $1/6 + 1/2 =$ $18 \times 6 =$ $327 \times 42 =$ $83 \div 7 =$ $253 \div 14 =$ Note: Students choose one or the other. One dot is easier and two dots is more challenging.	 Wai walks on a mile-long path near her house. On Monday she walked 0.3 miles. On Tuesday she walked 04 miles. Is her total walking closer to a half mile or a mile? Explain your answer. Wai walks on a mile-long path near her house. Every day she walks ²/3 of the path before going home. After five days, how far has she walked? Note: Students choose one or the other. One dot is easier and two dots is more challenging.
• Game to reinforce or investigate the big idea:	• Math Journal:	
NCTM Illuminations Fractions Game http://illuminations.nctm.org /activitydetail.aspx?id=18 ¹ > no more than four at a time > game only! The object of the game is to get all of the markers to the right side of the game board, using as few cards as possible. Click on the pile to turn over one card. This is your target fraction. Move the markers so that the sum of your moves is a fraction that is less than or equal to the target fraction. For example, if the first card turned over is ⁴ /5, you could move the fifths marker to ³ /5 and the tenths marker to ² /10, because ³ /5 + ² /10 = ³ /5 + ¹ /5 = ⁴ /5.	Post the following prompt: How many different fractions are the same (equivalent) to ¹ /4? How can you be sure there are that many? What makes them equivalent to ¹ /4? Remind the class that their journals must contain some answer and the evidence for it. If they don't have an answer, they should write about their current thinking and where they are stuck. (> Check on these before the end of the week. Be sure to meet with kids who have difficulty with this)	

¹National Council of Teachers of Mathematics, retrieved from: http://illuminations.nctm.org/activitydetail.aspx?id=18

Closure (about 10 minutes)	Ask:	
Closure (about 10 minutes) <i>Think:</i> <i>What question can I ask, or</i> <i>problem can I pose, to focus</i> <i>attention and discussion on the</i> <i>work from the last hour?</i> <i>What do I want students to</i> <i>remember about this lesson?</i> <i>How will I record student</i> <i>thinking?</i>	Ask: What strategies help compare two fractions? Give examples from your work today where you used these strategies. (If time, get some thinking about this) How might you compare ¹ /3 and ¹ /5? That creating equivalent fractions helps for comparing them. Use the SMARTboard capture for this discussion for the students who are out today.	



Main Lesson—Menu Lesson Plan Example: Seventh Grade

Main Lesson—Menu Lesson Plan BIG IDEA/FOCUS using unit rate to solve a proportion		
Explore/Main Lesson (about 20 minutes) Think: How can I provide opportunities for students to talk to each other about their math thinking during this lesson? How will I support all students to be part of this experience? Are the students doing the math work—or am I doing it for them?	 Inquiry or problem for the main lesson: Herbert buys his nectar from 3 different stores. Figway sells nectar for \$2.56/gallon. Tramway sells nectar for \$1.00/liter. BirdsRus sells nectar for \$1.00/quart. 1. Start by having groups discuss where there might be problems. 2. Have a whole-group discussion on where there might be problems and how groups are planning to attack them. If the unit of measurement issue comes up, tell them they can use the computer to find equivalence-or any other means they might have. 3. Solve and then report results and thinking. Closing problem to take to menu: How many hummingbirds could Herbert feed with that gallon of nectar? 	Ask: I want students to focus on the cost/ unit. It would be great if they found a common unit since this will allow them to compare the ratios.

Menu (about 30 minutes)

Think:

How will I differentiate this menu work for the variety of learners in my class?

How will I collect and respond to this work?

• Follow up from main lesson:	• Number and operations work:	• Problem to solve:
How many hummingbirds could Herbert feed with that gallon of nectar?	• $2/3 + 4/5 =$ •• $4/7 \div 1/3 =$ • $0.45 + 0.026 =$ $4^3/5 \times 1/2 =$ $3^{1/4} \div {}^{3/4} =$ $0.34 \times 0.2 =$ Put in order $1.25 \div 0.25 =$ $0.81, 1/8,$ $0.8, 8/100$ Note: Students choose one or the other. One dot is easier and two	 If 12 butterflies eat 60 grams of food, how much does each eat? If 4 butterflies eat 18 grams of food, how much does each eat? Note: Students choose one or the second each each each each each each each each
• Game to reinforce or investigate the big idea:	<i>dots is more challenging.</i>Math Journal:	other. One dot is easier and two dots is more challenging.
 Play Paper Pool (make sure materials are available) http://illuminations.nctm.org/Lessons /imath/Pool/pool1b.html' How to Play Paper Pool The lower-left corner is always corner A, and the labeling continues counterclockwise with B, C, and D. The ball always starts in corner A. The ball is hit with an imaginary cue (a stick for hitting a pool ball) so that it travels at a 45° diagonal across the grid. If the ball hits a side of the table, it bounces off at a 45° angle and continues its travel. The ball continues to travel until it hits a pocket.' 	Today we worked on helping Herbert buy nectar. You found out the cost of nectar in units. In what other ways do you use units to compare things? Think about shopping, traveling, and measuring.	

¹National Council of Teachers of Mathematics, retrieved from: http://illuminations.nctm.org/activitydetail.aspx?id=18

Closure (about 10 minutes)	Ask:	
Think: What question can I ask, or problem can I pose, to focus attention and discussion on the work from the last hour? What do I want students to remember about this lesson? How will I record student thinking?	How is finding the amount per measure related to multiplication or division? Do a pair/share before the final discussion. Finding the unit rate will help compare ratios. Use the SMARTboard. Save the class ideas to post on the class website.	If this is too challenging, try: When we found out which nectar was a better bargain, how did we use multiplication or division? How could we use them in problems that were like Herbert's problem?