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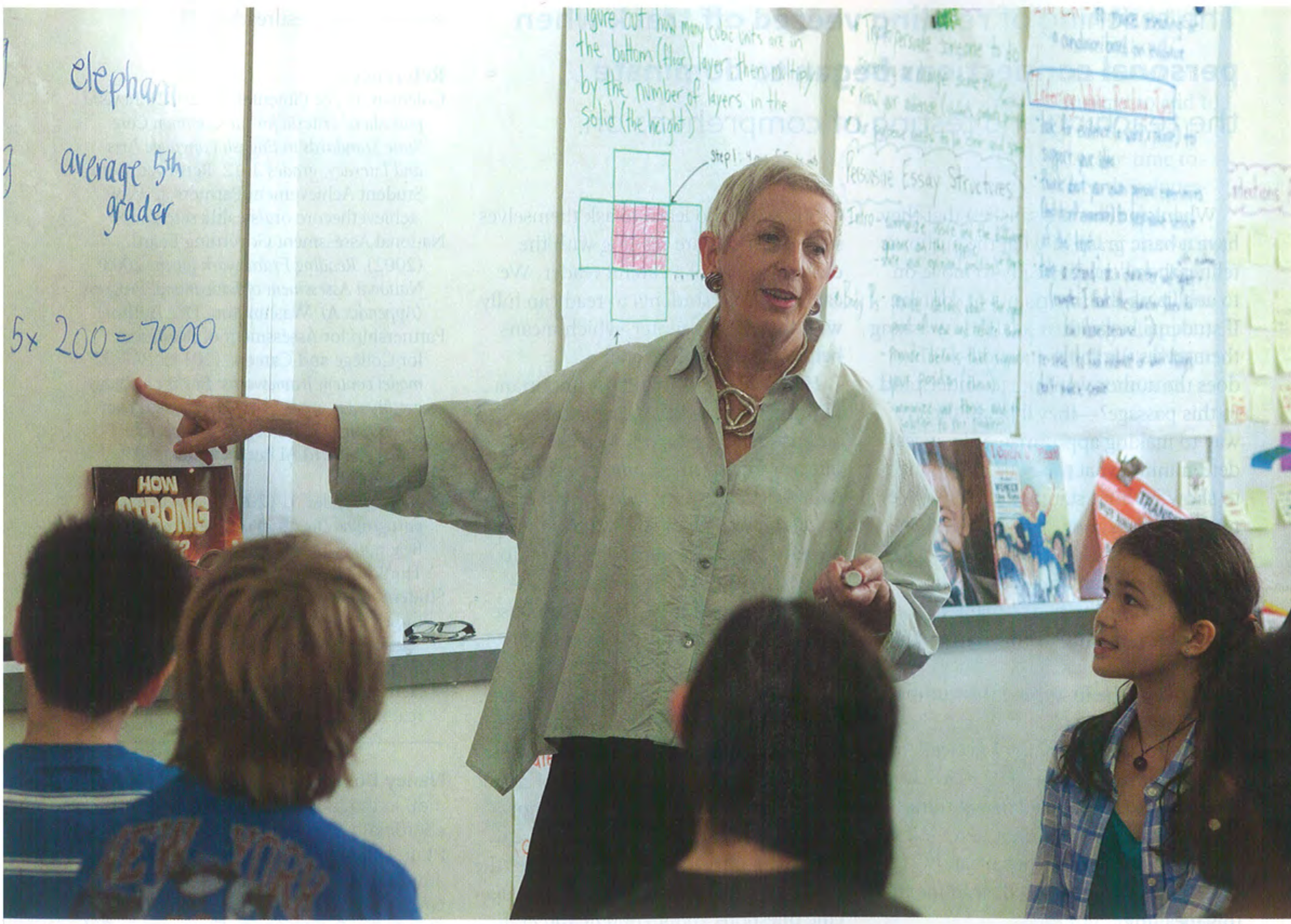
Go Figure

Math and the Common Core



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The Common Core State Standards for Mathematical Practice will require teachers to strengthen students' numerical reasoning and mental math skills.

Marilyn Burns

At a recent family dinner, my 11-year-old grandson, J. D., asked me how long ago I wrote *The I Hate Mathematics! Book*. I brought out a copy and asked him to check the copyright date, which was 1975.

"How long ago was that?" I asked. All of us around the table started to figure. After a few moments, we all agreed it was 37 years from 1975 to 2012.

"How did you figure out the answer?" I asked. (The math teacher part of me couldn't resist.)

J. D.'s 15-year-old sister, Hope, explained, "I knew it was 25 years to 2000, so I added 12 more and got 37." My husband Jeffrey did it the same way, but elaborated, "To add 25 plus 12, I added 10 and then 2." Everyone was nodding, except for J. D.

"That's not what I did," he said. His mother Deb encouraged him to explain.

J. D. said, "From 1975 to 2000 is 20 plus 5, and from 2000 to 2012 is 10 plus 2. So 20 plus 10 is 30, and 5 plus 2 is 7. It's 37."

It was quiet around the table. Then Hope said, "I don't get it." Jeffrey added, "That seems like a complicated way to figure."

"Tell me again," Deb said, trying to understand.

I was glad that Deb asked J. D. to explain again because I wasn't immediately clear about how he was thinking. I'd solved the problem the way Hope and Jeffrey did, and I had difficulty pulling myself away from the strategy we'd used to engage with the strategy J. D. had used, even as I could see that the way the rest of us solved it made little sense to him. I was reminded that one of the challenges of teaching is to listen to how students reason, rather than listening for responses we expect to hear.

Although I didn't initially understand J. D.'s reasoning, after thinking for a bit, I did—and I realized it was mathematically sound. Like the rest of us, J. D. used the year 2000 as a benchmark. He knew that it was 25 years from 1975 to 2000, and 12 more years from 2000 to 2012. To add 25 and 12, he decomposed each into its place value parts, added the tens (20 + 10) first, then added the ones (5 + 2), and then combined 30 and 7.

Math Talk and Common Core

At the time of this table talk, I was beginning to write this article about the Common Core State Standards and mathematics. I wanted to point out what's familiar in these standards and to give teachers clear access to what's different about them. I wanted to emphasize what has made me passionate about the Common Core standards—which is their two-part structure: Standards for Mathematical Practice and Standards for Mathematical Content, both equally important.

Let me start with a short description of these parts. The Standards for Mathematical Practice (referred to throughout this article as the practice standards) include the same eight standards for all grades (see "The Common Core State Standards for Mathematical Practice," p. 44). These practice standards describe the "expertise that mathematics educators at all levels should seek to develop in their students"—that is, the ways we want students to engage with the mathematics they're learning.[1]

In contrast, the Standards for Mathematical Content include many more standards, which are different for each grade. These content standards "define what students should understand and be able to do." They are organized into domains, each of which includes clusters of related standards so as to present mathematics as a subject of closely related, connected ideas. Teaching to the Common Core standards calls for making both the practice standards and the content standards integral to classroom instruction.

So how do the Common Core standards relate to J. D. and the problem of figuring out how long ago 1975 was? This is a traditional word problem, like those that have long been part of elementary school math. In the Common Core content standards, references to whole-number word problems appear at various grade levels. In grade 4, for example, under the Operations and Algebraic Thinking domain, one cluster of standards is "use the four operations with whole numbers to solve problems." In the Number and Operations in Base Ten domain, one cluster heading is "use place value understanding and properties of operations to perform multi-digit arithmetic." Although the Common Core language may differ from the language in various states' existing math standards, the word problem we solved at dinner illustrates a typical content expectation in all of them.

Examining my family's conversation as we solved this problem can help explain the spirit and intent of the Common Core practice standards. Figuring out how long ago the book was written called for making sense of a problem that required quantitative reasoning (Practice Standards 1 and 2). We presented our arguments for how we modeled the problem with mathematics (Standards 3 and 4). We were concerned with the precision of our answers (Standard 6). Some of us questioned how J. D. reasoned (Standard 3), and Jeffrey and J. D. explained how they broke numbers apart into their place value components (Standard 7).

Embracing the Common Core standards doesn't mean that it's essential to attend to every practice standard in all math lessons. All the Standards for Mathematical Practice are important, but different ones are appropriate at different times. For example, solving the problem in this anecdote didn't provide the opportunity to express regularity in repeated reasoning (Standard 8). Also, Practice Standard 5 calls for strategically using appropriate tools that students would have access to in the classroom, such as paper and pencil, concrete models, or calculators. None of these math tools were available at the dinner table, so we engaged in mental computation. Actually, computing in our heads exemplified the expertise called for in the practice standards.

The Common Core State Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Needed: Numerical Reasoning Skills

Assessing students' facility with numerical reasoning is essential to implementing the math standards. Although the Common Core standards are all about helping students make sense of math and become mathematical thinkers, arithmetic is still the cornerstone of the Standards for Mathematical Content through 5th grade. But even when teaching the basics of arithmetic—perhaps *especially* when teaching the basics of arithmetic—the Standards for Mathematical Practice should be at the forefront of math instruction. This means, for example, that students should be able not only to figure out the answer to a problem like 15 x 12, but also to demonstrate an understanding of multiplication as defined by the practice standards.

Solving problems mentally too often receives limited attention in classrooms compared with paper-and-pencil computation. Although developing written computation skills is important, mental computation helps develop facility with many of the practice standards—for example, reasoning quantitatively, constructing a viable argument, and looking for and making use of structure. Mental computation is also explicitly called for in the content standards. And it's an essential life skill—for example, when dividing checks at restaurants, deciding when to leave to arrive on time for appointments, adjusting recipes, or estimating how much you'd save if you bought something on sale. Figuring mentally is often sufficient—and more efficient than reaching for a tool.

The Math Reasoning Inventory (MRI), an online formative assessment tool I created with a team of colleagues with funding from the Bill and Melinda Gates Foundation, focuses on numerical reasoning (see "What Is the Math Reasoning Inventory?" on p. 46). Our charge was to help teachers assess how their students would respond to the kind of questions the Common Core standards expect entering middle school students to answer. The core of the assessment is one-on-one interviews in which students explain how they solved problems. The website (http://mathreasoninginventory.com) hosts video clips showing students' mathematical reasoning in action.

Watching these videos is helpful because observing students mentally solve math problems and explain their reasoning helps bring meaning to the practice standards. We're all familiar with what to expect when students solve 15 x 12 using paper and pencil. But what should educators expect to see when they ask students to figure out the answer to 15 x 12 mentally? What would be evidence of understanding and mathematical expertise as defined by the Common Core standards?

Following are descriptions of how four students, three 5th graders and one 6th grader, responded when asked to mentally figure 15 x 12. For each student, the assessor stated the problem aloud and showed it written on a card. After each student responded, the assessor asked, "How did you figure out the answer?" Each of these students arrived at the correct answer of 180, but in a different way. (See clips of these short interviews by going to https://mathreasoninginventory.com/Home/VideoLibrary and searching by the student's name, and then by the specific math problem.)

Monica's Reasoning

Just as J. D. decomposed 12 into its place-value parts to add it more easily to

25, Monica decomposed 12 into 10 + 2 and multiplied each part by 15. She explained, "I did 15 times 10, and it was 150. Then I did 15 times 2, which is 30, and it was . . . 180." Monica used the distributive property. Her method can be represented symbolically as 15 x 12 = (15 x 10) + (15 x 2) = 150 + 30 = 180.

Alberto's Reasoning

Alberto also used the distributive property but in a way that initially surprised me. He started with something he knew—12 x 12. He explained, "I did 12 times 12, [which is] 144, then I did 3 times 12 and I got 36, so I added 144 plus 36." If you watch this interview online, you'll see my surprise as Alberto explained his method. Alberto decomposed the first factor, 15, not into its place value parts, but instead into 12 plus 3. His method can be represented symbolically as 15 x 12 = (12 x 12) + (3 x 12) = 144 + 36 = 180.

Malcolm's Reasoning

Malcolm used the distributive property, but broke apart the 15 instead of the 12 so he could first multiply 10 times 12 and then 5 times 12. He explained, "I broke apart the 15 and then 10 plus 5, and then I did 10 times 12, which equals 120, and then I did 12 times 5, which equals 60, and then I added it all together and got 180." Malcolm's method can be represented symbolically as 15 x 12 = (10 x 12) + (5 x 12) = 120 + 60 = 180.

Cecilia's Reasoning

Cecilia used a very different method to figure out the answer. As she talked, she used her finger to "write" on the desktop to show how she was manipulating the numbers, as if using paper and pencil. Cecilia explained, "First, I'm breaking it into steps, and I'm doing 5 times 2. Then I leave the zero here and then I bring the 1 up here. Then 2 times 1 is 2, plus 1 is 3, so that's 30. And then 1 times 5 is . . . I mean, put a zero. Then 5 times 1 is 5, and then 1 times 1 is 1.

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So then the answer is 180."

Although Cecilia didn't have paper and pencil available, she simulated using them. Her method for this one problem doesn't necessarily indicate a lack of ability to reason numerically, but it would be a concern if she relied on algorithms as her only strategy for computing mentally.

It may seem surprising that students use different methods to solve the same arithmetic problem. Consider, however, that students usually write different topic sentences for the same writing assignment or that students in discussions often restate what someone else said in their own words, using alternate constructions to explain essentially the same idea. Why would we expect all students in math class to reason in the same way?

Notice how each of these learners exhibited the Common Core's practice standards. Each child persevered to solve a problem that called for quantitative reasoning (Standards 1 and 2). They each gave a viable argument (Standard 3); attended to precision (Standard 6); and used structure, breaking numbers apart to apply the distributive property (Standard 7—and a key understanding for algebra).

Natasha's Mileage Problem

It's also revealing to hear how another student, Natasha, reasoned through her solution to a different problem. The problem reads, "Molly ran 1.5 miles each day for 20 days. How many miles did she run altogether? "

In the clip, Natasha sits for a few moments looking at the problem written on a card. She hesitantly begins to explain: "I'm thinking that, I know that, in 2 days she ran 3 miles, because 1.5 times 2 would be 3 miles . . . 3 point zero. And 2 times 10 would be . . . ." Natasha stops, continues to think, scratches behind her ear, and then points again to the card, rereading the problem to herself. After being prompted to talk, she says, "OK, I know 2 days was 3 . . . 3 miles . . . and there's 20 days. So 2 times 10 is 20, so . . . ." Natasha pauses again to think, still looking at the problem on the card. After a moment, she says hesitantly, "30 miles?" She gets more confident as she constructs an argument that leads to the correct answer: "I got 30 miles because

I did 15 . . . 2 days is 3 miles, and 2 times 20 days . . . 2 times blank is 20, and I got 10, and 10 times 3 is 30 miles."[2]

I've shown this video clip to many teachers, and the reaction is always the same. Everyone is tense. Watching Natasha struggle is difficult, almost excruciating. Although the clip is less than two minutes long, it seems interminable to watch Natasha as she focuses on the problem. In this clip, we see evidence of almost all the Standards for Mathematical Practice (except for 5 and 8). Natasha makes sense of problems and perseveres, impressively, in solving them. She reasons abstractly and quantitatively and constructs a viable argument—although because this was a one-on-one interview, there's no opportunity for her to critique others' reasoning. She models with mathematics (using proportional reasoning); attends to precision (she corrects her own language several times); and looks for and makes use of structure.

What Is the Math Reasoning Inventory?

The Math Reasoning Inventory (MRI) is an online tool that helps teachers assess their students' numerical proficiency. MRI focuses on the numerical skills and understandings required by the Common Core State Standards for Mathematics for students entering middle school. There are three assessments in MRI—Whole Numbers, Decimals, and Fractions. Questions are based on content from the Common Core standards up to 6th grade. Each assessment includes an interview and a short written computation section. During the interview, which is the core of MRI, the teacher asks students to explain how they figured out answers.

The MRI website (https://mathreasoninginventory.com) provides, free of charge, complete instructions and support for administering these assessments. The site also enables teachers to generate individual and class reports that analyze their students' math reasoning. It provides specific information about the reasoning strategies and understandings required for numerical proficiency and includes a library of more than 80 videos that show students solving problems during interviews.

What We Learn From Listening

Analyzing Natasha's interview—as well as the other videotaped examples on the Math Reasoning Inventory site—shows the value of listening to students explain their reasoning. There is no way we would have learned so much about Natasha's use of the practice standards if she'd solved the problem independently with paper and pencil. The inventory is designed to assess students and then instantly provide a detailed report of the reasoning strategies and understandings that they do and don't demonstrate, which enables teachers to focus on strengthening any deficient reasoning strategies and underlying understandings about mathematics.

When teachers listen to students' math reasoning like this and realize that many students they're listening to can't demonstrate a particular strategy or understandings, instruction should change. Developing reasoning strategies and understandings should become part of the classroom culture, and at the forefront.

It's important not to think about "fixing" students who don't demonstrate particular skills or understanding, because partial understanding and confusion are part of the learning process—students learn in their own ways, at their own paces. But an assessment tool like the Math Reasoning Inventory can identify areas of weaknesses in the class and should guide us to teach in ways that reinforce students who are experiencing math success while giving more support to those who need it.

We should not judge students' numerical proficiency solely on their ability to compute with paper and pencil. We've all known students who can borrow, carry, and invert and multiply yet are unaware when their answers are unreasonable. These students typically lack numerical intuition and don't see the sense in mathematics. The challenge of the Common Core standards is to help all students develop enough mathematical expertise to be prepared for college or the workplace—and successful futures. EL

EL Online

For more on helping students master the Common Core State Standards for Mathematical Practice, see the online-only article "Instigating Thinking in Math Class" by Jessica Hiltabidel at www.ascd.org/el1212hiltabidel.

[1]National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core Standards for Mathematics.* Washington, DC: Authors. Retrieved from www.corestandards.org/Math.

[2]Natasha clearly meant "1.5" when she said "15." Students' language is sometimes imprecise even when their mathematical thinking is correct.

Author's note: The students shown are from a racially diverse school with a high percentage of low-income students and English language learners. Embracing the tenets of the Common Core initiative is a priority at the school; teachers have worked to help students develop the academic vocabulary they need to communicate mathematical reasoning.

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