Order of Operations

A Lesson for Grades 6–8

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This lesson was developed and refined by Math Solutions Education Specialists.

Lesson Objective

Students will understand the need for an agreed upon order in which to perform operations and apply the standard order of operations to a variety of number expressions.

Reference

- "Writing a PEMDAS Story," by Vadim Golembo (*Mathematics Teaching in the Middle School* 5, no. 9 [2000]: 574–79)

Materials

- Paper and pencil for each student
- Newsprint or chart paper, 1 sheet per pair of students
- Markers, 2 different colors per pair of students

Reviewed Vocabulary

punctuation, number expression, number sentence, order of operations, social conventions

Overview of Lesson

Students are introduced to two sentences that involve the same words. The meanings of the sentences are different because of a different use of punctuation. A number expression is introduced to the students that results in a variety of solutions based on the order in which operations are performed. A comparison between the word sentences and the number sentence provides a context for students to understand the need for an agreed upon order to perform operations. An agreed upon "meaning" or value for number sentences is as important as an agreed upon meaning for word sentences. Working in pairs, students use the standard order of operations to create a variety of number expressions that have specified values.

Lesson Outline

Focus or Warm-Up

1. Introduce the idea of using conventions to clarify meaning by displaying the following sentences:

   "Paul," said the teacher, "is very intelligent."

   Paul said the teacher is very intelligent.

2. Ask students to talk to a partner about the meaning of each sentence and be ready to share their ideas with the whole class.

   Important ideas to come from this discussion include how the words are the same but the punctuation changes the meaning. Explain that punctuation is something that has developed over time to help us communicate in writing.

Introduction

1. Next, ask the students to complete the following number sentence:

   $4 + 4 \times 4 - 4 =$ _____.

   Many students are unaware of the convention of the order of operations and the answers students offer might vary, including 28, 16, and 0. As students share how they arrived at their answers, record their thinking for the class. Some examples:

28	*16*	*0*
$4 + 4 = 8$	$4 \times 4 = 16$	$4 + 4 = 8$
$8 \times 4 = 32$	$4 + 16 - 4 = 16$	$4 - 4 = 0$
$32 - 4 = 28$		$8 \times 0 = 0$

2. Continue by discussing how confusing this could be when we are trying to communicate our thinking in writing. Explain that, just like in using agreed upon punctuation to clarify meaning with written words, we use an agreed upon order of operations to clarify meaning with written math sentences.

   Introduce the order of operations for the four basic operations:

   First: Simplify all operations inside parentheses.

   Then: Simplify all exponents, working from left to right.

   Next: Perform all multiplications and divisions, working from left to right.

   Finally: Perform all additions and subtractions, working from left to right.

3. Ask the students to again complete the number sentence but this time use the agreed upon order, or conventional order. Invite each student to compare his solution with that of another student sitting nearby. Finally, ask for students to share their solutions with the whole class. This time *most* of the solutions should be the same.

   Confirm the process and solution using the order of operations.

4. Refer back to the record of their thinking and model how to record their original thinking using number sentences. Use parentheses to communicate the order of operations used by each method:

28	*16*	*0*
$4 + 4 = 8$	$4 \times 4 = 16$	$4 + 4 = 8$
$8 \times 4 = 32$	$4 + 16 - 4 = 16$	$4 - 4 = 0$
$32 - 4 = 28$		$8 \times 0 = 0$
$(4 + 4) \times 4 - 4 = 28$	$4 + (4 \times 4) - 4 = 16$	$(4 + 4) \times (4 - 4) = 0$

Exploration

1. Next, introduce the *Four 4s* challenge. Note with students that in the lesson, three different values have already been created using only four 4s—28 and 16 and 0. Ask students to work with a partner to complete the challenge.

   Four 4s Challenge

   How many of the numbers from 1 to 10 can you create using four 4s?

   1 =

   2 =

   3 =

   4 =

   5 =

   6 =

   7 =

   8 =

   9 =

   10 =

   Record your findings on newsprint using dark markers and large lettering so others can read it from across the classroom.

   This challenge engages students in exploring how changes in operations affect the value of a numerical expression. In searching for expressions equal to given solutions, they develop strategies for manipulating the value of an expression and record expressions using conventional methods.

2. As partners complete their work, direct them to post their newsprint representations so they can be viewed by the whole class.

Summary

1. Ask the following questions as a way for students to demonstrate new learning and to reflect on the lesson objective.

   Q. What do you notice is the same about all of the newsprint posters?

   Q. What is different?

   Q. Which number sentences, if any, have the same value even though the operations were performed in a different order?

   Q. With which solution, if any, do you disagree? How might you correct it?

   Q. Why is it important that we have an agreed upon order of operations?

   Q. Who can state, using your own words, the standard order of operations?

   Q. The saying "Please excuse my dear Aunt Sally" has been used by many students to recall the standard order of operations. How do you think it helps?

Lesson Notes for the Teacher

Even though students can recite the correct order of operations, they need repeated opportunities to apply the order of operations in contexts that make sense.

A follow-up assignment might include writing a PEMDAS (parentheses, exponents, multiplication, division, addition, subtraction) story (see reference article by Vadim Golembo).

Here's an example from the article:

> *Problem:* Write a PEMDAS story about $(4 + 2) \times 2 \div 4$.
>
> *Possible Solution: Four friends were playing ball in the park. They were having a great day because it was the weekend. Later, two more of their friends from their neighborhood joined them. Now there were six friends playing in the park. Another group of six kids saw the group of six playing and asked if they could join to make two teams. Everyone agreed and now there were twice as many people playing; this made the game more competitive. Everyone was out to win. The group stayed in the park long after the game was over, just talking about their favorite topics.*
>
> *As it was getting later, everyone was getting tired and hungry. When they were ready to go home, the large group of twelve friends divided into four groups. Each group had the same number of people. This way four groups of three kids walked each other home.*

