3 Get to Zero

Overview

This activity gives students—individually, in pairs, or in small groups—practice with adding, subtracting, multiplying, and dividing whole numbers. Students can perform the calculations mentally or use a calculator, whichever you feel is more appropriate. Players start with a three-digit number and use any series of mathematical operations involving the numbers 1 through 9 to get to zero in as few turns as possible.

Materials Needed

A calculator for each student or pair or group of students.

Directions for Playing the Game

1. Players choose a three-digit number (example: 500).
2. Players choose an initial operation and number (example: divide 500 by 5); any number from 1 to 9 and any operation can be used. *Only whole numbers are allowed! If an operation results in a decimal answer, players must go back and try another number and/or operation.*
3. Players perform the calculation mentally or on the calculator and record the result on paper (example: 500 ÷ 5 = 100) so they can look back over their work.
4. Players repeat steps 2 and 3 until they get to zero.

Sample Game Scenarios

Turn 1: 500 ÷ 5 = 100
Turn 2: 100 ÷ 5 = 20

Turn 3: 20 ÷ 5 = 4
Turn 4: 4 – 4 = 0

Turn 1: 752 ÷ 4 = 188
Turn 2: 188 ÷ 2 = 94
Turn 3: 94 – 4 = 90
Turn 4: 90 ÷ 9 = 10
Turn 5: 10 ÷ 5 = 2
Turn 6: 2 – 2 = 0

Extensions

1. Give everyone the same number to start with and challenge the class to get to zero in as few operations as possible.
2. Ask students to find as many three-step numbers (those from which you can get to zero in only three operations) as they can.

IN THE CLASSROOM WITH RUSTY

Introducing the Activity

"I'm going to teach you an activity called Get to Zero," I began. "You'll need a calculator and a piece of paper and a pencil. You may work alone, with a partner, or with a small group."

The sixth graders in Pam Long's class had each been assigned a calculator the first week of school, and everyone now eagerly pulled it from his or her desk. To help model the directions, I used Pam's overhead calculator, as well as a projected transparency on which I could record numbers.

"To begin, you need to choose any three-digit number," I explained. "Once you've chosen your number, write it down on your paper. Would anyone like to suggest a three-digit number for us to work on together?"

"How about 500," offered Nancy.

I wrote *500* on the transparency. "Our goal is to get to zero in as few mathematical operations as possible," I said. "You may use any operation: addition, subtraction, multiplication, or division. When using one of those operations, you may only use the numbers one through nine. I'll work through an example. What operation should we begin with?"

"I think you should start with division because it gets you a smaller number than all the other operations," said Carl. Students nodded their head in agreement. Although Carl's idea was obvious to these sixth graders, the comment signaled a sense of numbers and operations.

I then proposed dividing 500 by five. I chose the number five because I knew it wouldn't yield an answer with a remainder. I wanted to keep things uncomplicated while modeling the activity.

"I'll begin by dividing 500 by five," I stated. "What's 500 divided by five?"

"One hundred!" the students chorused.

Then I punched the numbers into the overhead calculator, encouraging students to verify the answers using the calculators at their tables. Calculating mentally not only often is faster than a calculator and makes more sense but also helps develop mathematical thinking.

I wrote *divide by 5* under the number *500* on the transparency before I continued. This was turn number one.

"Now I'll divide by five again," I told them. "I want you to mentally divide 100 by five."

After a few moments, several students raised their hand. I asked the class quietly to say the answer to 100 divided by five. Then we verified the answer on the calculator to be sure it was 20. Again, I wrote *divide by 5* for turn number two.

"Now we have 20 showing on the calculator," I said. "We want to get to zero in as few moves as possible, so what should we do?"

"Divide by five again!" exclaimed Manuel.

"Or divide by four," added Mary.

"What about divide by ten?" asked Michael.

"That wouldn't work, you can only use one through nine," said Todd, reminding Michael of the rules.

"Think about what would get us the smallest quotient, or answer," I suggested. After a few seconds, several hands flew up. I called on Carl.

"If you divide 20 by five you'll get four, but if you divide it by four you'll get five," he said. "So I think we should divide by five."

"I don't think it matters," Gordon interjected. "Because either way you'll get to zero in the same number of turns."

"Gordon, can you tell us more about that?" I asked.

"Well, if you divide 20 by five, the answer's four and then you could subtract four to get to zero," he explained. "If you divide 20 by four you get five and all you have to do is subtract five to get to zero. For both, you get to zero in two turns."

"Does that make sense?" I asked. Students nodded their head.

"We could choose four or five as a divisor," I said. "Let's try five and divide 20 by five."

"It's four," said Hannah.

"Okay," I responded. "In this game, it's handy to use the calculator to keep track of what's happening to the numbers as we make our way to zero." After I verified the answer on the calculator, I wrote *divide by 5* for turn number three. "What should we do next?" I asked.

"Subtract four!" several students chimed in.

I finished by writing *subtract 4* for turn number four. "So it took us four turns to get to zero," I said. "Let's try another one. Raise your hand if you have another three-digit number less than 1,000 for us to begin with."

"Let's start with an odd number," suggested Blanca. Blanca often challenged the group and she exuded confidence when working with numbers. "How about 123?"

I wrote *123* on a new transparency. "I want you to talk with your neighbor about what operation and number we should use to begin," I said. After a moment, I asked for volunteers and called on Xavier.

"Divide 123 by two," he suggested.

Several students groaned and others shook their head in disagreement.

"Before we divide 123 by two, I'm interested in hearing what you think will happen," I said.

"I think the answer is going to be a number with a remainder," Cam predicted.

"Why do you think that?" I asked.

"Because two will go into 12 evenly, but two won't go into three evenly," he responded. Cam appeared to be solving 123 divided by two using the long-division algorithm.

"I think you won't end up with a whole number and you have to have a whole number for an answer or else it's really hard to get to zero," said Jenny.

"When you're dividing, how do you know if you'll get an answer that's a whole number?" I asked.

"You just have to have the feeling for what's going to divide evenly into a number," Kerry mused. "If you don't know, then you'd have to play around with the numbers until you get a whole number."

"Let's use our calculators and divide 123 by two," I instructed. Students soon realized that dividing by two resulted in an answer with a remainder.

"I got 61.5," Cam reported. "That's the same as 61 and a half. If you get a decimal, it's hard to get to zero."

"So it's important that you end up with a whole number for an answer whether you divide, multiply, subtract, or add. I think Xavier's idea helped us learn new things about this activity," I said. "How about another idea?"

"Let's divide 123 by three," offered Hannah. "I'm picking divide by three because 123 is a multiple of three."

Students used their calculators to divide and came up with 41 as an answer. I continued to keep track of the operations and numbers we were using on the projected transparency, modeling for students how they might keep track of their decisions. (Figure 3.1 shows how Niqueta kept track of her games.)

"What next?" I asked.

"We should subtract one so we get to an even number," said Devin. "That would give us 40."

I wrote *subtract 1* on the transparency. That was our second operation. I asked students to think about what to do next and reminded them that we wanted to get to zero in

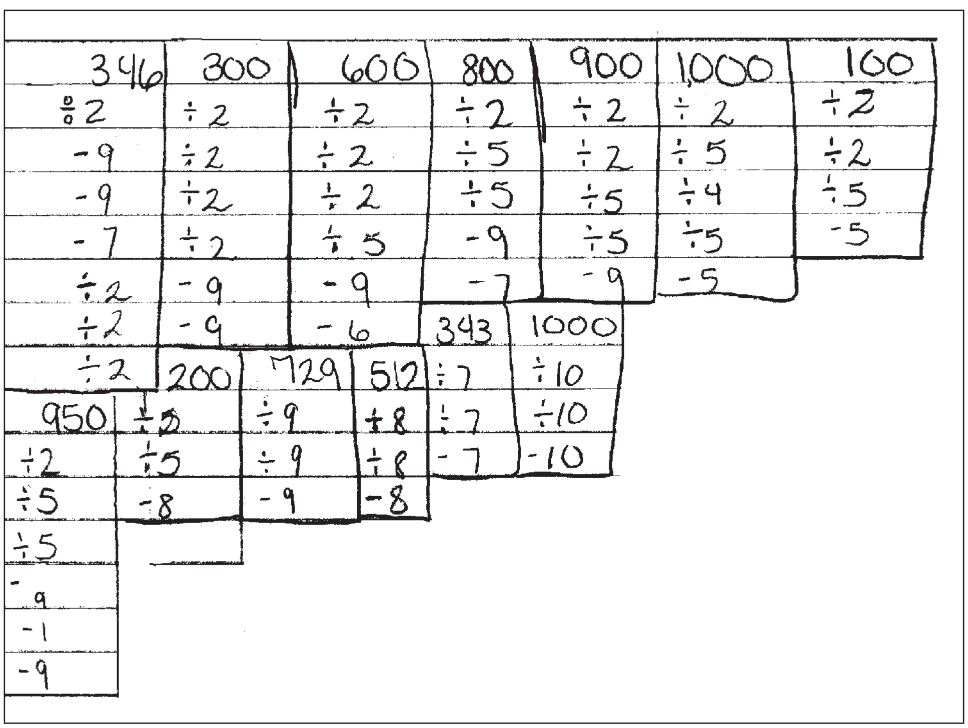


FIGURE 3.1

Niqueta's record of her Get to Zero games.

as few operations as possible. Someone suggested dividing 40 by two, but that idea was vetoed in favor of dividing by five. Students were beginning to make sense of the game. We divided 40 by five to get eight, then subtracted eight to get to zero in four turns.

Observing the Students

After we finished this second game together, students got to work; some partnered up and some worked alone. The game was motivating and sustained the students' interest. I moved around the room, mostly observing and asking questions.

I noticed that being able to use the calculators freed the students to explore numbers and operations. They seemed uninhibited and challenged by trying to get to zero in as few operations as possible.

A Writing Assignment

After about twenty-five minutes, when it seemed that most students had explored several sequences, I asked them to write about the game. I provided these prompts:

- This game helps me learn . . .
- I discovered . . .
- I think . . .
- I found out . . .

A Class Discussion

After giving students about ten minutes to write, I called the group back together for a class discussion. "Raise your hand if you'd like to tell us about something you discovered," I said.

"The fastest way to get to zero is by using division," reported Katie.

"If you're like at 443 or something, you want to subtract to have your number end in zero, so you can divide by five easily," said Anne.

"Tell us more about that, Anne," I probed.

"Well, if you're at 443 and you subtract three you get to 440 and you can then divide by five evenly, because I know that numbers that end in zero are multiples of five," she explained.

"What else did you discover?" I continued.

"That, like 555, 444, 333, all come out the same way," said Carl.

"Give us an example, and I'll write the numbers on the chalkboard," I told him. "Everyone else use your calculator and follow along with us."

"Five hundred and fifty-five divided by five equals 111, divide by three equals 37, subtract seven equals 30, divide by six equals five, subtract five equals zero," he reported. "It works for every three-digit number where the digits are all the same. You get to zero in five turns." (Carl's written work is shown in figure 3.2.)

"Let's try another one," I said. "Let's test Carl's conjecture."

"Four hundred and forty-four divided by four equals 111, divide by three equals 37, subtract seven equals 30, divide by six equals five, subtract five equals zero," he said carefully. The

I discovered that if you choose a # like 100 200 300 ect and you divide it by the hundr-ed # example 200÷2 or 300÷3 youll get 100 nomatter what then you do the 3 steps for 100 and that is 100÷5 ÷5 −4 and you have a total of 4 steps example 200÷2=100÷5=20÷5=4 −4=0 I also discovered # like 555 and 444 and 333 all take 5 steps to do example 555÷5÷3÷7−6−5, 444÷4÷3÷7−6−5 you divide those #s by their # like on 444 is 4 and youll get 111 example 555÷5=111 666÷6=111 777÷7=111 then it takes you 4 steps to get rid of 111 so a total of 5 steps.

FIGURE 3.2

Carl discovered a pattern with certain numbers.

class seemed impressed by this discovery.

"Any other discoveries?" I asked. Lots of hands shot up. I was pleased that this activity had stimulated so much thinking.

"I learned that if you have a hundred number, if you divide it by the number in the hundred's place, then you will always get 100," said Katie. "Like 800 divided by eight equals 100, 900 divided by nine equals 100, like that. I think that's important, because when you get to 100, you can get to zero in three turns." (Katie's written work is shown in figure 3.3.)

"Remember when we got to 100 in three turns when we played as a class?" I reminded them. "Did anyone get to zero in less than three turns?"

"If you divide by two digits, you could," said Anne. "But we can't do that."

"Did anyone else get to zero in three moves?" I asked.

"I got to zero in three moves starting with 200," Hannah reported. I wrote the equations on the chalkboard as she read them from her paper. "Two hundred divided by five is 40. Divide 40 by five and that's eight, then subtract eight," she read.

"How about 512," said Denny. "Five hundred and twelve divided by eight equals 64, 64 divided by eight equals eight, then subtract eight."

Get To Zero

The game get to Zero helps you learn faster ways to get to a certain number Like if you had three 9's in a problem and the question was what would the total be if you multiplied the numbers together, you wouldn't have to multiply them all you would just know by playing this game. I learned that if you have a hundred number if you divide the number in the hundred's place by it then you will always get 100. Like 800 ÷ 8 = 100. I also learned that dividing is the fastest way to get to zero

FIGURE 3.3

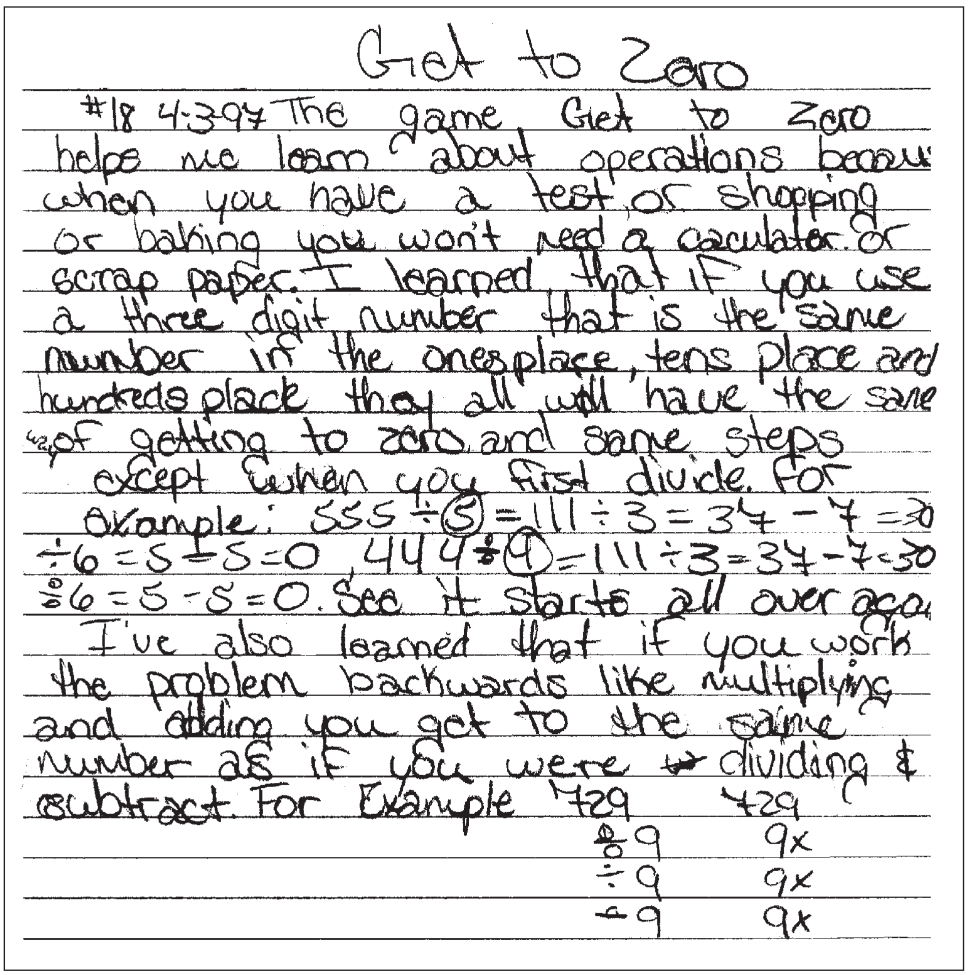
Katie realized that dividing is the fastest way to get to zero. She also discovered how to get to 100 quickly.

"Did anyone get to zero in less than three moves?" I asked. No one raised a hand.

"I think it's impossible," was Jeremy's conjecture.

"What's the largest number we can start with and get to zero in three moves?" I asked. Jenny was waving her hand so hard I thought it was going to fly off. She was smiling excitedly.

"I got 729!" she exclaimed. "I worked backwards. I multiplied nine times nine times nine and that's 729. I multiplied by nine because that's the highest number you can use with the operations. When you start with 729, you divide by nine and that equals 81. Divide 81 by nine and that's nine, then subtract nine." (Jenny's written work is shown in figure 3.4.)



Get to Zero

#18 4-3-97 The game Get to Zero helps me learn about operations becau when you have a test, or shopping or baking you won't need a caculator or scrap paper. I learned that if you use a three digit number that is the same number in the ones place, tens place and hundreds place they all will have the same of getting to zero and same steps except when you first divide. For example: 555 ÷ 5 = 111 ÷ 3 = 37 − 7 = 30 ÷ 6 = 5 − 5 = 0, 444 ÷ 4 = 111 ÷ 3 = 37 − 7 = 30 ÷ 6 = 5 − 5 = 0. See it starts all over aga

I've also learned that if you work the problem backwards like multiplying and adding you get to the same number as if you were dividing & subtract. For Example

729 ÷9 ÷9 −9

729 9x 9x 9x

FIGURE 3.4

Jenny noticed that Get to Zero helped her learn more about operations. She also explained how to get from 729 to zero in three moves.

"I was working with Jenny and I tried her way with eights," said Katie. "I did eight times eight times eight and got 512, then I got to zero in three turns!"

"Sixes work too!" Carl announced. He'd been experimenting on his calculator while Katie was explaining her discovery. "You multiply six times six times six and then you get 216 to start with."

"Would fives work?" I asked. Everyone began multiplying mentally to check this out. They were on a roll.

"That would work, 'cause the answer is 125 and it's still a three-digit number," said Brennan. Brennan was usually disengaged and uninterested in numbers. He struggled with math, but now he was right there with us.

"What about fours?" I asked. "Will four times four times four get us a three-digit number?"

After a few seconds, several students chorused, "No!"

"Why is that?" I asked. I knew that many students in the class knew why, but there are always some who are on the periphery of the conversation, not quite following or understanding. I wanted to give those students a chance to listen to an explanation. I called on Carl.

"Because four times four is 16 and 16 times four is 64," said Carl. "And 64 is not a three-digit number, so we can't start with it."

"Does anyone else have a discovery they'd like to share?"

"This game helps me learn to divide at the right time," said Kerry. "Like when the number is 111, I would know to divide it by an odd number, because 111 is an odd number. If I would divide 111 by two, then it wouldn't come out evenly, because two is an even number. So I think that if the number you're working with is even, then divide it with an even number and if it is odd, then divide it with an odd number." (Kerry's written work is shown in figure 3.5.)

"We're almost out of time, but I think Kerry's conjecture deserves some thought," I told the class. "Kerry, can I write down on the board what you just said so that we can think about it?" I made a point of getting Kerry's permission before opening her conjecture up to scrutiny. Sharing mathematical ideas is risky. I think it's important to maintain an atmosphere of respect during class discussions so that children feel safe expressing their ideas.

On the chalkboard, I wrote: *In this game, divide an odd number by an odd number and an even number by an even number*. "Is this what you mean?" I asked Kerry, pointing to the words. She nodded yes. "Talk about Kerry's statement with someone in your group," I told them. After several minutes, I asked for everyone's attention and called on Michael.

"I think that Kerry's partly right," said Michael. "I think that you can't divide an even number into an odd number evenly and an odd number into an even number evenly. But just because you start with an odd number and divide it by an odd number doesn't mean there'll be no remainder. Like nine divided by five doesn't work."

"Or seven divided by three," added Katie.

"I think you have to think about

Dividing

This game helps me learn to divide
at the right time. Like when the
number is 111 I would know to
divide it by a odd number cause
111 is a odd number. If I would
divide 111 by 2 then it wouldn't
come out evenly. Because 2 is an
even number. So I think that if the
number your working with is even
then divide it with an even #.
If it is odd, then divide it with
an odd number. So I think that
strgety will help me, to find out what
to divid with what.

FIGURE 3.5

Kerry's faulty conjecture prompted a rich class discussion.

the multiples of a number when you're dividing," said Hannah.

"Kerry's conjecture made us think more about numbers, especially how they're used in division," I said, bringing our class discussion to a close.

RUSTY ANSWERS YOUR QUESTIONS

How does this activity help students develop their number sense?

Facility with computing is an important characteristic of number sense. Get to Zero gives students practice with all the operations, especially division. And using calculators allows them to take risks and try new ways of thinking.

This activity gives students opportunities to think about operations and what happens to quantities when they're added, subtracted, multiplied, and divided. During the activity, for example, Carl revealed his knowledge about division in relation to the other operations when he commented that "you should start with division because it gets you a smaller number." When Anne recommended subtracting till your number ends in zero so that you can divide it by five evenly, she was drawing upon her understanding of the multiples of five and of division. She was using her number sense to think about a strategy.

Get to Zero also gives students the chance to explore the characteristics of numbers: odd and even, factors, multiples, prime and composite. In addition, students are able to learn about decimal numbers, look for patterns, and make conjectures and test hypotheses, all of which help develop their number sense.

How can Get to Zero be modified so that it's more easily accessible?

One way to make this activity more accessible is to start with numbers between 50 and 100 instead of any three-digit number and to use the digits one through five instead of one through nine. Working with smaller numbers is less daunting for students and is therefore a more comfortable place to start.

A colleague of mine taught Get to Zero to her class using only the numbers 50 through 100 and the digits one through five. Then she asked her students to describe strategies they thought were helpful in getting to zero. Here are the strategies they came up with:

1. Choose even numbers.
2. Try to divide by the biggest number you can.
3. If you can't divide, then add or subtract to get to a number that ends in five or zero.
4. Never divide when you have an odd number.
5. If you have an odd number, subtract one, three, or five to get an even number.

Would students need a calculator at home if Get to Zero were a homework assignment?

Having a calculator at home is not necessarily a prerequisite. In this game a calculator is used only to verify computations and to keep track of the numbers as students make their way to zero. Students could certainly play Get to Zero without a calculator.

Although students primarily solve the problems in Get to Zero mentally, they benefit from learning how to use a variety of tools to solve math problems or explore mathematical ideas. A calculator is an important tool that should be available to students in math class.

How would you help students become aware of the divisibility rules?

While some students discover rules for divisibility on their own, it helps to make them explicit so that all students have access to them. Still, students will need a good deal of experience to become comfortable with the divisibility rules. Encourage them to look for patterns. Some are easy to recognize, such as counting by fives and noticing that all the multiples end in zero or five, or learning that all even numbers are multiples of two. But the discovery that if the sum of the digits in a number is a multiple of three (for example, 123, or 1 + 2 + 3 = 6), then the number is a multiple of three is not as obvious.

The benefit of a game like Get to Zero is that it provides a reason for students to think about divisibility, another valuable way to understand relationships among numbers.