



# MARILYN BURNS: MENTAL MATH

Figuring out answers in our heads is an important skill. Give it a starring role in your math teaching.

By Marilyn Burns

Every day in the grown-up world, we face situations that call for adding, subtracting, multiplying, or dividing. We figure tips in restaurants, decide when to leave home to get to the movies on time, estimate the price of a sale item, keep track of what we're spending while shopping in the supermarket, double and halve recipes, and on and on. At least half of the time, we do these calculations when we're out and about, without pen and paper handy, which means we have to figure mentally. Because figuring in our heads is such an important life skill, it should have a regular role in your classroom math teaching.

## MATH WITHOUT PENCILS

When I want to work on mental math skills with students, we do what I call "hands-on-the-table math." It's a simple idea. Ask students to clear away everything and place their hands on the table. No paper and pencil, no manipulatives, no books. Then, present a problem for them to solve in their heads. Here's how:

To support the children and to be sure this is a learning opportunity—not a testing situation—make solving the problem a class effort. (For details, see the sidebar on page 3.) Have students share their ideas and hear from their classmates. Meanwhile, guide the instruction and record the students'

## Explaining Mental Math to Kids

*Here's how I explain the guidelines for hands-on-the-table math to my students.*

■ **WE USE OUR HEADS.** Hands-on-the-table math means that we don't use paper, pencil, or any other materials to solve the problem.

■ **WE DO IT TOGETHER.** You won't have to figure out the answer by yourself. We'll solve the problem together.

■ **WE NEED EVERYONE'S HELP.** Your job is to be willing to share your ideas and to listen to everyone else's ideas.

■ **WE RECORD ON THE BOARD.** My job is to listen to everyone's ideas and help us remember them by keeping track of them on the board.



ideas on the board. Recording gives you the opportunity to model for the students how to represent their thinking mathematically. When you do this, you'll find that students quickly become comfortable with the routine of hands-on-the-table math. Then you can use it for all or part of any math lesson to present a variety of mental math challenges.

### MENTAL MATH IN ACTION

All sorts of problems are appropriate for hands-on-the-table math, as long as they can be solved mentally. Even problems that don't seem immediately suited for mental math can be turned into hands-on-the-table problems. For example, a multiplication problem like  $148 \times 21$  isn't appropriate if the goal is to find the exact answer. But presenting it as an estimation problem is a good challenge for fourth or fifth graders. Try writing a problem like this on the board:

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Which answer is closest to the product?

$148 \times 21 =$  1000

2000

3000

4000

5000

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Give students a few moments to think quietly by themselves. Then ask them to turn and talk with a partner or small group. Lead a class discussion, calling on students to share ideas. Encourage them to explain their thinking. Represent the ideas mathematically on the board. For example, if a student offers that  $148$  times  $10$  is  $1480$ , so  $1000$  can't be closest, write " $148 \times 10 = 1480$ " on the board.

Continue the discussion until all students have shared ideas and understand why  $3000$  is the closest estimate. Even if students share the same idea, it's good for them to explain it in their own words.

As a follow-up assignment, ask students to explain, on paper, the solution to the problem. Students might choose to do the multiplication to find the exact answer. They might also refer to what

you recorded on the board.

### GETTING TO THE RIGHT ANSWER

For some problems, an exact answer is the goal. Asking first or second graders how they would figure out the sum of two single-digit numbers,  $6 + 7$ , for example, is a good way to focus students on different strategies for addition. Or give an addition problem with two two-digit numbers,  $26 + 57$ , for example, and have students explain how they find the sum. To provide variety with addition practice, give students a two-digit number and have them figure out how much more is needed to make  $100$ . Or give them a larger number,  $578$  or  $1,375$ , and ask them how they could rename it as the sum of two smaller numbers, or, if appropriate, as the sum of three smaller numbers. Even though all these problems call for exact answers, there are multiple ways for students to figure out solutions. From hearing others' ideas, students broaden their own repertoire for computing, which helps support their number sense and builds flexibility.

Some problems don't require students to do one particular calculation, but engage them in multiple calculations to test different possibilities. For example, list seven different quantities and ask students to come up with a number sentence that uses three of them. Here is a sample that uses money as a context:

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A number sentence uses three of the following:  $\$1.50$ ,  $2$ ,  $\$3.75$ ,  $50$  cents,  $6$ ,  $\$3.00$ ,  $75$  cents. What could the number sentence be? One possible solution is  $\$3.00 \div 2 = \$1.50$ .

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Money provides a useful context for many problems. Here are some problems that have multiple correct solutions.

- How could you spend exactly  $\$100$  by buying two things with different prices?
- How could you spend exactly  $\$100$  by buying three things with different prices?
- How could you spend exactly  $\$100$  by buying three things with different prices if one of the items costs  $\$39$ ?

Some problems don't require computing, but still encourage interesting discussions that build students' understanding about numbers.

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A number has been rounded off to 1,200. What could the number be?

Or:

I wrote down a number with one zero in it, but I can't remember what it was. I know it was between 500 and 800. What could the number be?

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For all problems, encourage students to explain their reasoning, even when they give correct answers. Too often students learn that teachers only ask them to explain when they're wrong. In hands-on-the-table math, explaining is important for all students all the time.

## KEEPING TRACK OF MATH THINKING

Too often, students see paper-and-pencil work merely as time for them to practice skills. There is a time for paper-and-pencil computation practice, for sure, but it's valuable to think about paper-and-pencil work more broadly, too. Students' writing should also be a vehicle for them to keep track of their thinking. The recording students do as a result of hands-on-the-table math supports this.

## MENTAL MATH IS TEST PREP

To help prepare students for standardized tests, most teachers distribute practice booklets. Next time, try hands-on-the-table math instead. Write a problem on the board, but don't write the choices for answers. Instead, conduct a hands-on-the-table discussion until the class agrees on a solution. Then post the answers. Follow this with a discussion of the other possibilities that weren't correct, and talk—again with hands on the table—about what sort of errors could lead someone to choose an incorrect answer. Your students will be amazed at their mental power! □

## Marilyn answers your questions



**Q.** *What if my students don't offer any ideas to solve the problem?*

**A.** Having students first talk about the problem with a partner or group helps avoid this. However, if the students are still reticent when you begin the class discussion, ask who can present an idea that someone else had. Sometimes it's less risky for students to offer how someone else was thinking.

**Q.** *What do I do when a student suggests an answer that is wrong?*

**A.** Don't interrupt or correct the child immediately. Many times, students self-correct if given the chance. Sometimes if you restate what a student says, recording it on the board will make the error obvious. Or another student might have an objection. Of course, you don't want to end a lesson with a misconception still existing, but don't rush into resolving problems. Let the discussion evolve.

**Q.** *What if a problem is just too hard for the class?*

**A.** Move on to another problem. There's always a similar question that can help you address the same concept or skill. This happened to me recently when I read Mem

Fox's book *Night Noises* to a class of third graders. The main character in the book, Lily Laceby, celebrates her 90th birthday, and I asked the children to figure out the year she was born. We checked the copyright date of the book and found it was written in 1989. The children were stumped; the problem was simply out of their reach. I shifted the problem, asking the children to think about what year she would have been born if she had just turned 90 this year. The children were able to dive into this version.

**Q.** *When I ask students to solve the problem with paper and pencil, won't they simply copy what's on the board?*

**A.** It's fine for them to rely on the numerical representations on the board. Since they have to explain their reasoning, they have to choose something from the board that makes sense to them. Ask students to make their responses as detailed as possible so that you can see what they do and don't understand.

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MARILYN BURNS IS THE FOUNDER OF MATH SOLUTIONS PROFESSIONAL DEVELOPMENT. FOR IDEAS ABOUT QUESTIONS TO ASK YOUR STUDENTS, CHECK HER WEB SITE ([WWW.MATHSOLUTIONS.COM](http://WWW.MATHSOLUTIONS.COM)) FOR THE MATH SOLUTIONS PUBLICATIONS RESOURCE, *GOOD QUESTIONS FOR MATH TEACHING: WHY ASK THEM AND WHAT TO ASK*.