Introducing Multiplication of Fractions

A Lesson for Fifth and Sixth Graders

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Teaching multiplication of fractions is, in one way, simple—the rule of multiplying across the numerators and the denominators is easy for teachers to teach and for students to learn. However, teaching so that students also develop understanding is more demanding, and Marilyn Burns tackles this in her new book Teaching Arithmetic: Lessons for Multiplying and Dividing Fractions, Grades 5–6 *(Math Solutions Publications, 2003). In the following excerpt from Chapter 2, Marilyn builds on what students know about multiplying whole numbers to begin developing understanding of what occurs when we multiply fractions.*

I began the lesson by posting the chart of "true" statements about multiplying whole numbers that I had previously generated with the class:

> *1. Multiplication is the same as repeated addition when you add the same number again and again.*
>
> *2. Times means "groups of."*
>
> *3. A multiplication problem can be shown as a rectangle.*
>
> *4. You can reverse the order of the factors and the product stays the same.*
>
> *5. You can break numbers apart to make multiplying easier.*
>
> *6. When you multiply two numbers, the product is larger than the factors unless one of the factors is zero or one.*

I planned to use these statements as a base for helping the students think about multiplying fractions. To begin, I pointed to the first statement:

> *1. Multiplication is the same as repeated addition when you add the same number again and again.*

"Do you think this is true when we think about fractions?" I asked. I wrote on the board:

$$6 \times \frac{1}{2}$$

"Talk with your neighbor about how you might make sense of this problem with repeated addition," I continued.

After a few minutes, I called on Juanita. She said, "I think you can do it by adding one-half over and over again. I did one-half plus one-half, like that, six times. I think the answer is three." I wrote on the board:

$$6 \times \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 3$$

"How did you get the answer of three?" I asked.

Juanita responded, "One-half plus one-half is one whole, and you can do that three times, and you get three." I wrote:

$$6 \times \frac{1}{2} = \underbrace{\frac{1}{2} + \frac{1}{2}}_{1} + \underbrace{\frac{1}{2} + \frac{1}{2}}_{1} + \underbrace{\frac{1}{2} + \frac{1}{2}}_{1} = 3$$

Eddie added, "It's like if you had six times something else, you could add the something else six times, and that's what Juanita did with the one-halves."

"So, does this first statement work for multiplying with a fraction?" I asked. The students nodded their agreement, and I wrote OK next to the first statement.

I then pointed to the second statement:

2. Times means "groups of."

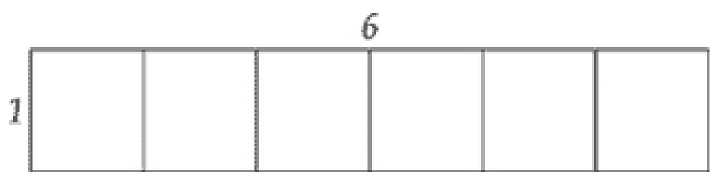
"Does it make sense to read 'six times one-half' as 'six groups of one-half'?" Most of the students nodded.

Saul added, "The answer is still three." I wrote *OK* next to the second statement and then pointed to the third statement:

3. A multiplication problem can be shown as a rectangle.

I asked, "Can we draw a rectangle to show six times one-half?" The students weren't sure.

"Suppose the problem were six times one," I said, writing *6 x 1* on the board. The students were familiar with using rectangles for whole number multiplication. I sketched a rectangle, saying as I did so, "One side of the rectangle is six units long and the other side of the rectangle is one unit long." I labeled the sides *6* and *1* and then divided the rectangle into six small squares.

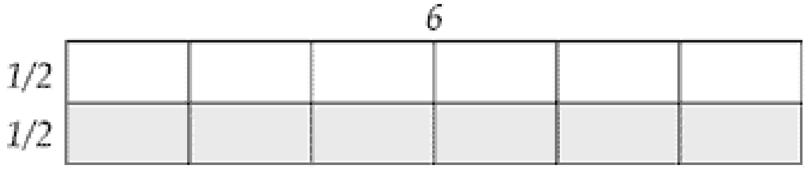


"See if this rectangle helps you think about how I might draw a rectangle to show six times one-half," I said.

Kayla said, "Just cut it in half."

"Which way should I cut the rectangle?" I asked.

"Sideways," Kayla said. I split the rectangle as Kayla suggested, then erased the 1 and replaced it with $\frac{1}{2}$ written twice. Also, I shaded in the bottom half to indicate that it wasn't part of the problem.



"The top half of the rectangle is six units by one-half unit and shows the problem six times one-half. The bottom shaded half shows the same problem again, but we don't need to consider both. How many squares are there in the unshaded rectangle? Does this still give an answer of three?"

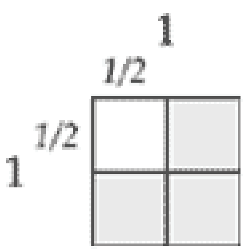
Damien explained, "Two halves make a whole, and you do that three times, so the six halves make three whole squares. Three is still the answer."

"But what about if both the numbers are fractions?" Julio challenged.

"Let's try one," I said. I wrote on the board:

$$\frac{1}{2} \times \frac{1}{2}$$

I decided to show the students a way to think about representing the problem with a rectangle. "When I draw a rectangle for a multiplication problem with fractions, I find it easier first to draw a rectangle with whole number sides. So, for this problem, I think about a rectangle that is one by one," I said. I drew a square on the board, labeled each side with a *1*, and continued, "This rectangle is a square because both factors are the same. It shows that one times one is one. Now watch as I draw a rectangle inside this one with sides that each measure one-half." I divided the square, shaded in the part we didn't need to consider to show the $\frac{1}{2}$-by-$\frac{1}{2}$ portion in the upper left corner, and labeled each side $\frac{1}{2}$.



I said, "The part that isn't shaded has sides that are each one-half of a unit. How much of the one-by-one square isn't shaded?"

"One-fourth," several students responded.

Saul was skeptical. He asked, "You mean that one-half times one-half is one-fourth?" I nodded.

"I don't get it," he said.

"But do you agree that the unshaded rectangle has sides that are each one-half?" I asked. Saul nodded.

"But you're not sure that the answer of one-fourth is correct?" I asked. Again, Saul nodded.

"Let's see if the other statements can help you see why," I said. I knew that if the students thought of the problem as "one-half of one-half," they would agree with the answer of one-fourth. I planned to develop this idea, and I used the next statement to do so.

I pointed to the next statement:

4. You can reverse the order of the factors and the product stays the same.

Craig commented, "That should work." Others agreed.

"But it doesn't matter for one-half times one-half," Brendan said. "If you switch them, you still have the same problem."

I said, "Right, so let's think about this statement for the first problem we solved—six times one-half. What about if we think about the problem as one-half times six?" I wrote on the board:

$$\frac{1}{2} \times 6$$

I continued, "If we think about the times sign as 'groups of,' then one-half times six should be 'one-half groups of six.' But that doesn't sound right. It does make sense, however, to say 'one-*half of a group* of six,' or 'one-half of six,' and leave off the 'groups' part. Both sound better, and they're still the same idea. What do you think 'one-half of six' could mean?"

"It's the same," Sabrina said. "One-half of six is three, so one-half times six is three, and that's the same as six times one-half."

I said, "Let's think about one-half times one-half the same way. What is one-half of one-half?"

I heard several answers. "A fourth." "A quarter." "One-fourth."

"So what do you think about reversing the order of the factors when the factors are fractions?" I asked.

The students agreed that it would work, so I wrote *OK* next to Statements 3 and 4.

"Let's look at the fifth statement," I said, pointing to it:

5. You can break numbers apart to make multiplying easier.

"Talk with your neighbor about how you could apply this statement to the problem six times one-half."

After a few moments, I called on Brendan. He said, "It works. You could break the six into twos, and then you do two times one-half three times. Two times one-half is one. One plus one plus one is three. So it works." I wrote on the board:

$6 \times \frac{1}{2}$

$6 = 2 + 2 + 2$

$6 \times \frac{1}{2} = (2 \times \frac{1}{2}) + (2 \times \frac{1}{2}) + (2 \times \frac{1}{2})$

$= 1 + 1 + 1 = 3$

Anita said, "We split the six into four and two. Half of four is two and half of two is one and two plus one is three. It works." I recorded:

$6 = 4 + 2$

$4 \times \frac{1}{2} = 2$

$2 \times \frac{1}{2} = 1$

$2 + 1 = 3$

I wrote OK next to the statement. "We have one statement left," I said, pointing to it:

> *6. When you multiply two numbers, the product is larger than the factors unless one of the factors is zero or one.*

I asked, "Does this statement hold true for six times one-half?"

"It doesn't work," Sachi said. "Three is smaller than six, so it doesn't work."

"Could we change the statement so that it does work?" I asked.

Julio said, "It should say at the end 'unless the factors are zero or one or a fraction.'" I edited the statement as Julio suggested:

> *6. When you multiply two numbers, the product is larger than the factors unless one of the factors is zero or one or a fraction.*

I thought about how to respond. If one of the factors is a fraction less than one, then Julio's idea works. But if the fraction is more than one, Julio's idea might not work. I wanted to acknowledge Julio's idea but also give him the chance to refine it. I posed a problem that had a fraction as one of the factors for which the answer was greater than both of the factors. I said, "That's a good idea, but I think it needs a little more information. Think about this problem—six times three-halves. That's the same as six groups of three-halves." I wrote on the board:

$6 \times \frac{3}{2}$

$\frac{3}{2} + \frac{3}{2} + \frac{3}{2} + \frac{3}{2} + \frac{3}{2} + \frac{3}{2}$

"Talk with your neighbor about what the answer would be to this problem," I said. After a couple of minutes, I called on Craig.

"We got nine," he said. "We knew that three-halves is the same as one and a half, and one and a half plus one and a half is three, and three plus three plus three is nine, so the answer is nine." I recorded Craig's idea on the board:

$$6 \times \frac{3}{2}$$

$$\frac{3}{2} + \frac{3}{2} + \frac{3}{2} + \frac{3}{2} + \frac{3}{2} + \frac{3}{2}$$

$$\frac{3}{2} = 1\frac{1}{2}$$

$$\underbrace{1\frac{1}{2} + 1\frac{1}{2}}_{3} + \underbrace{1\frac{1}{2} + 1\frac{1}{2}}_{3} + \underbrace{1\frac{1}{2} + 1\frac{1}{2}}_{3}$$

$$9$$

Craig continued, "And nine is bigger than six or three-halves. So the statement doesn't work. It works the way it used to be, but it doesn't work the way you changed it."

"I think I can fix what I said," Julio said. "The fraction has to be smaller than one. So any number that is zero or one or in between makes an answer that is smaller than the factors."

"So what should I write to change the statement?" I asked.

Julio said, "At the end, write 'zero or one or a fraction that's smaller than one.'" I wrote on the board:

> *6. When you multiply two numbers, the product is larger than the factors unless one of the factors is zero or one or a fraction smaller than one.*

Julio and others nodded. I wrote *OK* next to the revised statement.

This was just the first lesson I planned to teach about multiplying fractions. In later lessons, I further developed the students' understanding and skills.