One Thousand Paper Cranes

A Lesson with Sixth, Seventh, and Eighth Graders

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In One Thousand Paper Cranes *(Dell Laurel-Leaf, 1997), Takayuki Ishii writes a nonfiction version of the story of Sadako Sasaki, which became well known in America through a novel and picture books by Eleanor Coerr. In 1955 Sadako died of leukemia, caused by exposure to radiation from the atomic bomb dropped in 1945 on Hiroshima. Sadako thought that if she could make one thousand cranes, her wish to stay alive would come true. Her story so touched the hearts of others that her image appears at the top of the Children's Peace Statue in Hiroshima Peace Memorial Park.*

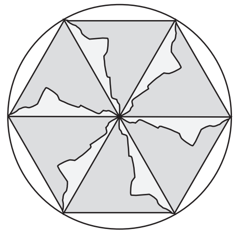
This lesson, excerpted from Math and Nonfiction, Grades 6–8 *(Math Solutions, 2009), focuses on geometry concepts. After reading excerpts from the story to her class, Jennifer M. Bay-Williams has her eighth graders create an asymmetrically shaped bird whose image they use to perform rigid transformations on a coordinate plane. Specifically, students use the* x- *and* y-*axes to reflect the nonsymmetrical image of their origami bird and draw conclusions about the relationship of the coordinates of the original shape to those of the reflected shape.*

Prior to this lesson, I prepared a transparency of grid paper with coordinate axes in the center and a transparency of folding instructions for the paper crane we would be making (see blackline masters at the end of this lesson). My students had used diagrams to do some simple paper folding and therefore knew how to do basic origami folds. If your students do not have experience with origami, I recommend selecting any basic origami construction as a beginning experience prior to doing this investigation. These can be found through an online search or in the many available origami books, such as *The Joy of Origami,* by Margaret van Sickle (Workman, 2005). I also made sure that several graphing calculators were available for those students who wanted to use them.

I began the lesson by asking students, "Who has a design that has sixty-degree rotational symmetry?" The day before this lesson, I had given them the following assignment, for which they had used a protractor.

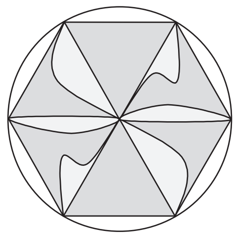
> *Using a circle, inscribe a hexagon by measuring angles. Once the hexagon is drawn, create a design on the template that will have 60° rotational symmetry.*

As I held up each volunteer's design, I asked the other students if they agreed.



The design is the same
at a 60 degree rotation.

When students disagreed, I had them work in pairs to decide what kind of symmetry the design did have. Brea's design did not have 60-degree rotational symmetry, but it did have 180-degree rotational symmetry:



This design is repeated
at 180 degrees.

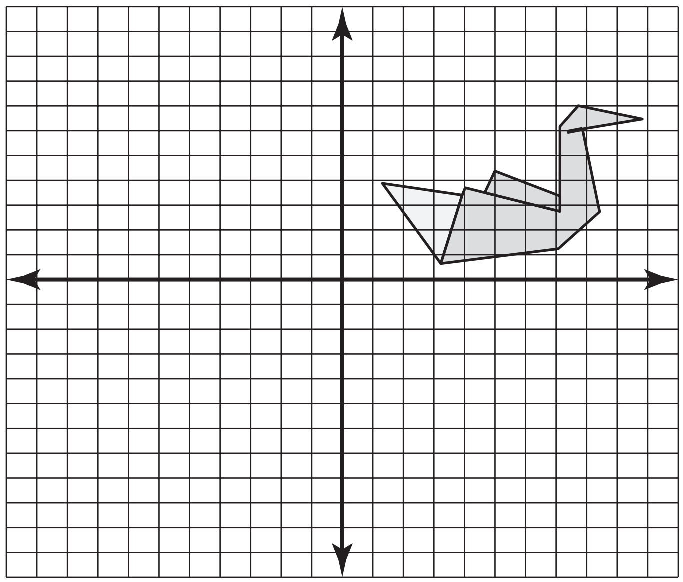
Next I held up the book *One Thousand Paper Cranes* and asked, "How many of you know the story of Sadako?" A few students shared what they knew. I then read aloud passages of the book about Sadako's creation of the cranes (pages 58–60) and the creation of the Children's Peace Statue (pages 74–81). The class listened intently.

I asked if any of the students had made origami birds before. Several had. To date, however, they had folded paper to make figures that had symmetrical designs; the crane they were about to make had no symmetry and involved more complex paper folding. I shared the instructions for making the crane by placing the transparency on the overhead projector.

After giving two sheets of 6-by-6-inch origami paper to each student, I asked everyone to create the crane. I also asked the students to see how many different types of triangles and other polygons they noticed in the shape as they worked. Making the crane takes some time and is of medium difficulty, and it could be done as homework instead of in class.

Once students had finished their birds, I asked, "What symmetry does the bird have?" Students noted the "aerial view" had line symmetry. I asked students to look at the side view of the crane and see if its shadow had any symmetry. It is not symmetrical; therefore, it is a good shape for introducing transformations on a coordinate plane. A nonsymmetrical figure lends itself to a clear interpretation of whether the image has been rotated, reflected, or translated.

Next, I reviewed the transformations we were going to be performing with the crane. I placed a transparency of the coordinate axes on the overhead projector and placed my bird (facing to the right and lying on its side) in Quadrant I of the coordinate plane. I said, "Here is our flattened crane. If we reflect it over the *y*-axis, what will it look like?"



Angela said, "It flips so that the head is looking to the west instead of to the east."

"What if we put it back in Quadrant One and rotate it over the *x*-axis?" I continued.

"It would be upside down," several students replied.

Joe added, "It kind of looks like the crane seeing its reflection in a pond."

"Very interesting connection!" I commented. "What if we translated the crane by moving it to the left and down?"

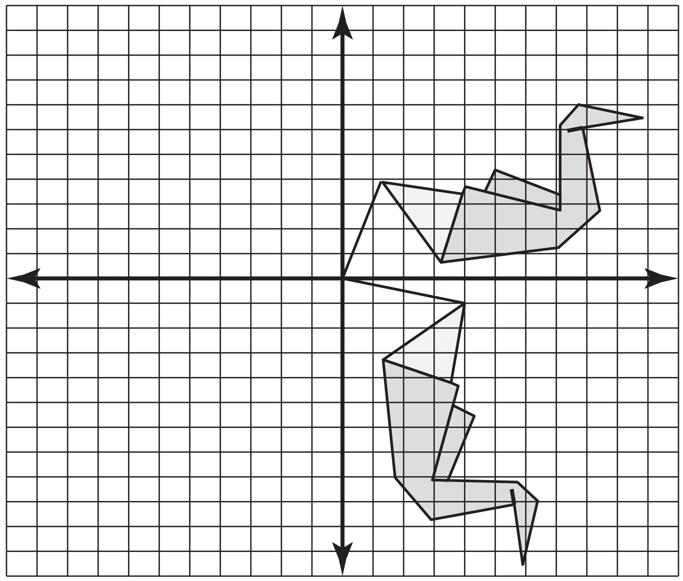
"It would be in a different spot, but facing the same way," Taquita said.

"What if I rotated the crane by ninety degrees?" I asked.

"It would turn," Brian said.

"Elaborate on that, Brian," I probed.

Brian approached the overhead and illustrated how the crane would turn and be sideways. He turned it so that it rotated into Quadrant IV, as shown below.



"Remember, with rotations, we need to know where our point of rotation is so that we can rotate at that point. Brian has picked the origin as his point of rotation," I added.

I said to the class, "These transformations are called *rigid* transformations. Why are they called rigid?"

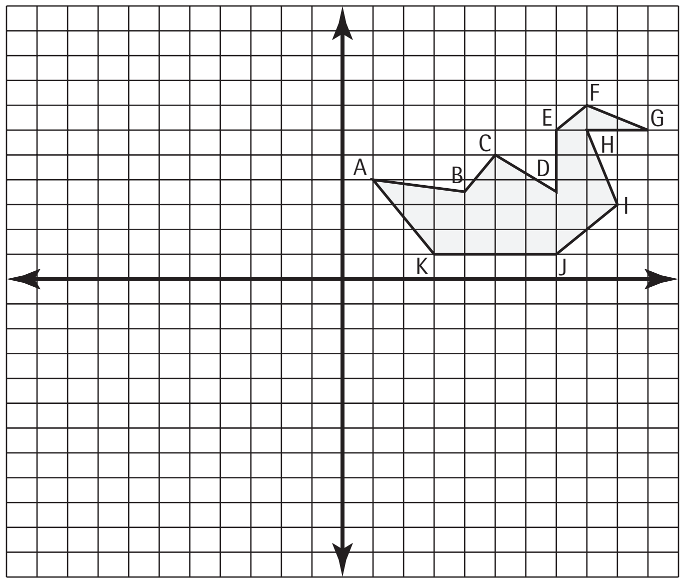
Stephanie said, "Because the shape of the crane doesn't change."

"What do you mean it doesn't change?" I probed.

Miranda raised her hand and said, "Well, the position can move, but the shape itself doesn't change. So, the crane will be the same shape even after it's been moved."

I agreed, "Yes, the shape of our bird is the same as we move it around."

I modeled for students how to mark the points that made the vertices of the bird and then connect the dots. Students could now see the preimage (original position) of the crane on the transparency even when the bird was lifted from the grid. With student input, I labeled the coordinates of the bird's vertices with letters and then wrote the coordinates in a list.



A = (1, 4)	E = (7, 6)	I = (9, 3)
B = (4, 3.5)	F = (8, 7)	J = (7, 1)
C = (5, 5)	G = (10, 6)	K = (3, 1)
D = (7, 3.5)	H = (8, 6)	

Next, I explained to students that they would be doing two reflections: one over the *y*-axis and one over the *x*-axis. As they worked, they were to think about and be ready to share a rule for how the coordinate pairs were affected by each reflection. I wrote the following on the board:

How will reflections affect the coordinate pairs of a figure?

What patterns do you notice with coordinate pairs when you

1. *reflect the crane over the* y*-axis?*
2. *reflect the crane over the* x*-axis?*

I replaced the crane in Quadrant I and asked students, "What will I do as a first step in exploring Question One?"

Many hands went up. Samuel replied, "You are going to flip it backwards, to the left." I asked him to come up and show me what he meant. He came to the overhead, picked up the bird, flipped it over the *y*-axis, set it down, and returned to his seat.

I asked the rest of the class, "Is this OK?"

Taquita said, "Does it matter where it goes in Quadrant Two, or does it just have to be backwards?"

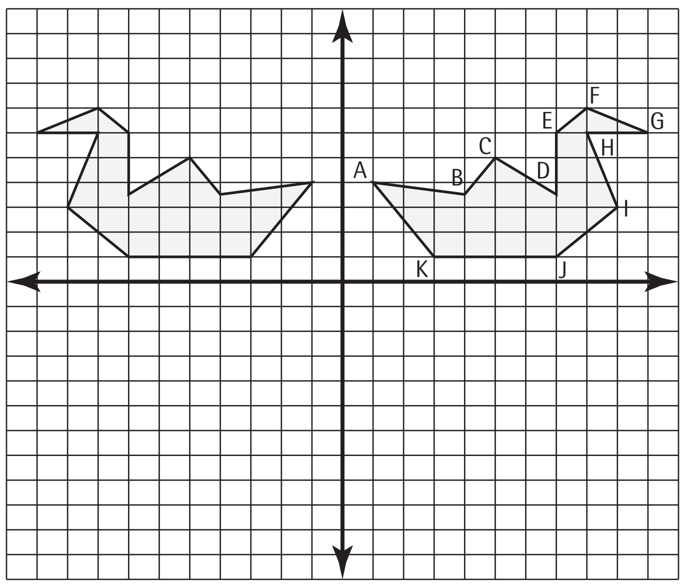
"Great question," I said. "What is the answer?"

"It can go anywhere, as long as it is lined up," Felipe offered.

"I think it has to be the same distance from the *y*-axis," Celina stated.

I asked students to think about these two points: "For a reflection, does the object need to be lined up, straight? Does the position have to be exact?"

"It has to be exact," Jeremy chimed in, "because the *y*-axis is like a mirror. If we fold the paper on the *y*-axis, it has to match up." To model the last idea, I folded the transparency over the *y*-axis, traced the bird, and opened the transparency to show what a reflection would look like.



The class was quiet. I asked if the students agreed and they all did. I asked, "Can someone explain the next reflection?"

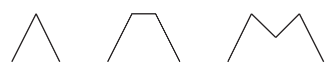
Brian said, "So if you reflect over *y*, it will be backwards. If you reflect over *x*, it will be upside down. So if we do the two, will it be in the third quadrant?"

I clarified that each time they did a reflection, they needed to start with the preimage, or original image, in Quadrant I. Then I said to the class, "This raises a good question. If we perform two transformations, one over each axis, what would the image in Quadrant Three look like? Would it be the same if we went to Quadrant Two first as it would be if we went to Quadrant Four first?" The students weren't sure. I suggested they start with the reflections into Quadrants II and IV and then see what happened if they reflected over the axis into Quadrant III.

I distributed half-centimeter grid paper, one sheet to each student, and had the students draw a set of axes in the center. I reminded them to record all of their coordinate pairs and to keep looking for relationships between the preimage and the new image they were making.

I asked them to let me know if they thought they saw a relationship. In addition, because I thought some students would confuse the x-axis and y-axis, I labeled a rough sketch on my transparency that showed that the bird reflected over the y-axis was in Quadrant II and the one reflected over the x-axis was in Quadrant IV.

Students worked in pairs, picking one of the birds they had made and using it to do each transformation. I noticed one pair of students was sketching the original bird in Quadrant I when one of them asked, "What do we do with the vertices that don't end up on an exact point?" I responded that they could round to the nearest half unit. Another student wanted to know if the tip of the wings should be represented with one point, a flat line, or two points:



We decided that it would be fine to do any of these sketches, as long as the original bird and its images had the same shape, since we were doing rigid transformations. We agreed it would not be OK if the original image had one point and the reflection had a flat line.

Another pair of students had flipped the crane and were recording points, but their bird was crooked. I asked, "If the original bird has a vertex at the point (one, six), then what should the position be after flipping it?"

Karissa said, "The opposite."

Her teammate, Hannah, added, "It would also be six high, but on the other side."

"So it would be (negative one, six)?" Karissa asked as she moved her bird to be on that coordinate and recorded the point. I asked how they might check if they were doing the reflection accurately. They each took their coordinate plane and folded it to see if the points in Quadrants I and II matched up.

Some of the students who were having trouble finding the points by using their bird decided to make a template of the bird, creating a flat image. They traced the bird onto colored paper and cut it out.

I reminded students to consider where the points were in Quadrant I as they worked on their sketches. Some students labeled the points with the coordinate pair. Others numbered the vertices and then recorded the coordinate pairs in a list they made in Quadrant III.

As I observed, I could see that some pairs continued to discuss and compare what coordinates they recorded and some pairs worked more independently. A number of the students continued to use the crane as a guide to make sure they were placing each point in the correct place in the new quadrant.

Amanda used a ruler to make sure the distance from the axis was the same for each reflected point. To do the transformation over the *y*-axis, she placed her ruler at the height of the vertex and then counted the number of lines to get to the new point and marked the spot. She continued this until she was finished with all the coordinate pairs.

Jackson did not use a ruler to plot the points but counted the vertical lines and then used the bird as a check to see that the shape was still preserved.

After students completed their sketches, several started to enter their data into their graphing calculators. I stopped the class for a moment and encouraged students to do this only after they had finished all the recording of their reflections over each axis. The Stat Plot feature allowed the students to program the calculator to assign Plot 1 so that L1 and L2 were *x* and *y* coordinates, Plot 2 so that L3 and L4 were *x* and *y* coordinates, and Plot 3 so that L5 and L6 were *x* and *y* coordinates. They entered the *x* and *y* values for the original bird in columns L1 and L2, then entered the coordinates of the bird that was reflected over the *y*-axis in L3 and L4, and, lastly, entered the coordinates of the bird reflected over the *x*-axis in L5 and L6 (the last two steps can be switched).

The calculators enabled students to confirm that the coordinates they had recorded in fact would preserve the bird's shape and be reflected appropriately. (See Figures 1 through 3.)

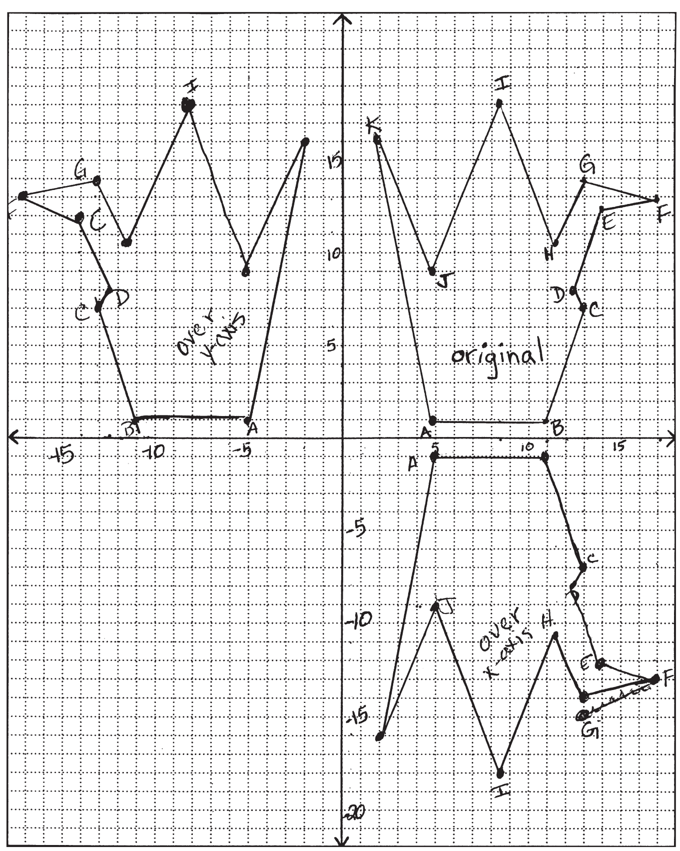


Figure 1. One student's reflection.

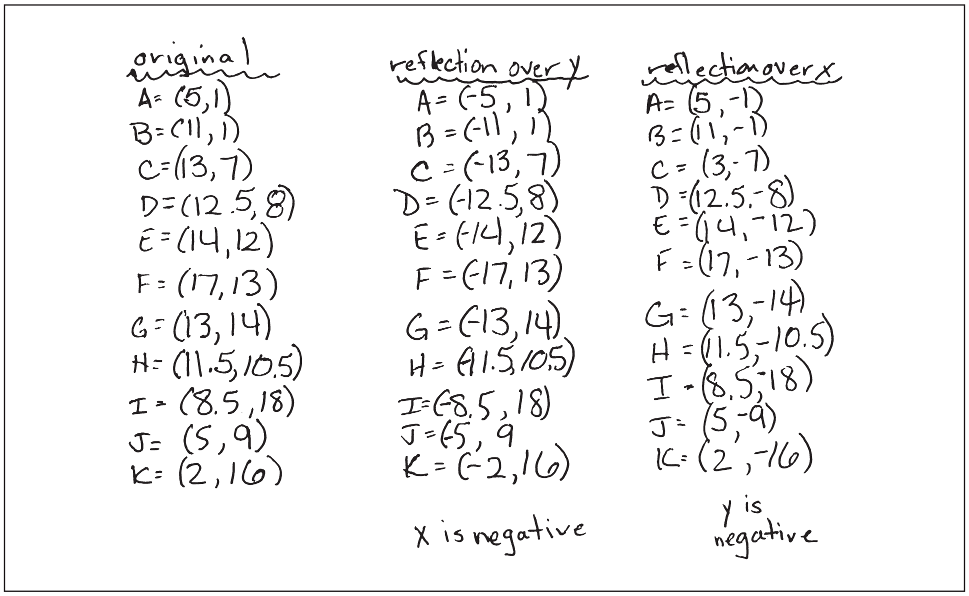


Figure 2. Coordinates for the reflections in Figure 1.

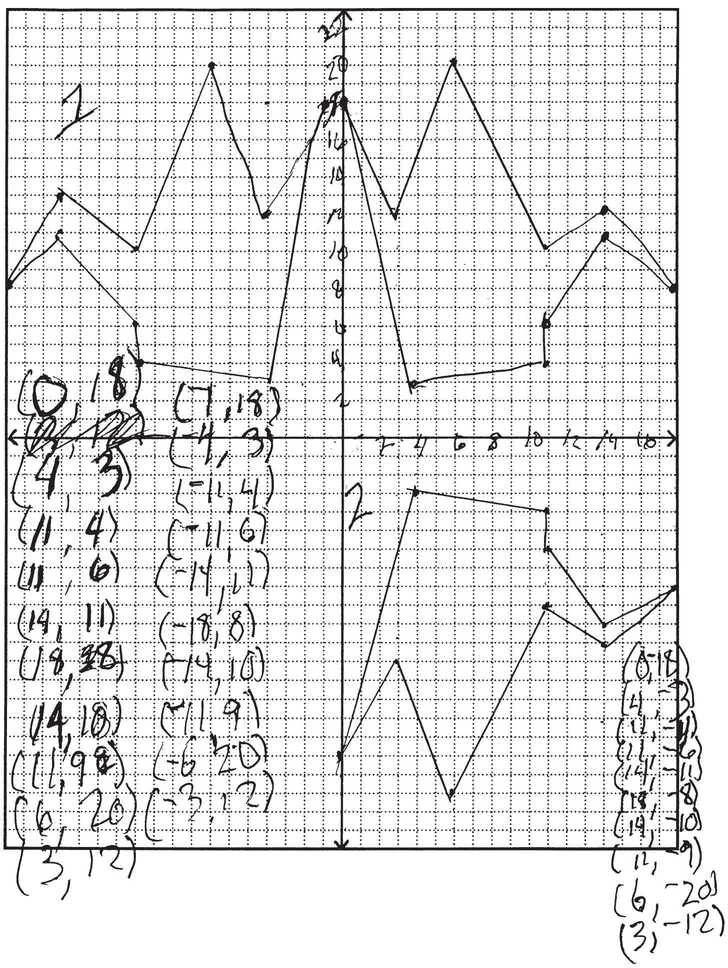


Figure 3. This student listed all his coordinates on the graph.

When everyone was ready, I led a class discussion. I asked students to explain what they noticed about the coordinates when they reflected the bird over the y-axis. Aaron said, "The y stays the same and the x value is the opposite."

"When does y stay the same?" I asked for clarification.

Aaron replied, "When it is flipped over the y-axis, the y stays the same."

"Why is this?" I asked. "Talk to your partner for a minute."

After pairs had some time to share, I asked the question again.

Katrina said, "It is because, if you take one point, like the beak, it is the same y, the same height; you are really just moving to a new x."

I asked, "Does anyone else have another way to explain this?"

Amanda, who had used a ruler, said, "It's like placing the ruler parallel to the x-axis and then finding the point on the other side of the y-axis, so the y doesn't change—it's the same."

I asked students to record what this meant in algebraic terms. I wrote on the board:

> *Reflecting over the* y*-axis:*
>
> (x, y) → _____

Trevor offered, "It would be negative x and y." I recorded:

> (x, y) → (–x, y)

I asked partners to talk this over and see if they agreed. In pairs, there was some discussion about which coordinate became negative, but they confirmed Trevor was correct by looking at their own data.

"What about reflecting over the x-axis?" I asked. "Work with a partner and look at your reflections over the x-axis and let me know what you think the reflection rule is and how you would record it algebraically."

Modeling after the reflection over the y-axis, students recorded:

> (x, y) → (x, –y)

I asked, "How could your rules be used if we were to do this task again?"

At first the students were not sure what I was asking, but then Amanda said, "We wouldn't need to draw the pictures for the reflections; we could just use the rules and find the new coordinates."

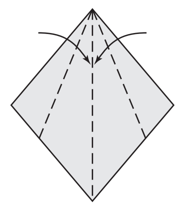
This was an aha moment for several students, who said, "You mean we could have used this rule instead of drawing?!" I reminded them, while pointing at the instructions on the board, that I had started the lesson by encouraging them to keep their eyes out for a pattern that could help them find the coordinates for a reflection. It was a nice opportunity to reinforce that keeping our eyes out for rules or generalizations can help us solve problems more efficiently.

Note: For additional lessons that use nonfiction as a springboard, see *Math and Nonfiction, Grades 6–8*, by Jennifer M. Bay-Williams and Sherri L. Martinie (Math Solutions Publications, 2009).

How to Make a (Basic) Origami Bird

1. Place your paper with the colored side facedown. (This will give your bird a white body but colored head, neck, and wings.)

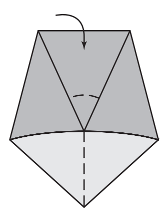
2. Fold your square along a diagonal and then unfold it, leaving it flat. The plain side should still be face-up. Turn the square to a diamond position with the crease line running from top to bottom, then fold the left- and right-side corners in to the center fold and down so the left and right sides meet along the center fold line.



3. At this point the paper should look like an upside-down kite—broad at the bottom and narrowing toward the top.

4. Fold the top corner down to the spot where the left and right corners now meet. This fold makes the bird's neck.

5. Fold the end of that fold back up a little bit. Don't worry about exactly how far you fold it. This smaller fold will be the bird's head.



6. Fold the bird in half away from you (with the colored side facing down) along the crease you made in Step 1. (Make sure all previous folds remain intact.)

7. Pull the neck out from the body so the neck and head are vertical to the body. Make a new crease at the base of the neck.

8. Pull the beak up, and create a new crease on the back of the head.

9. You can pull the wings out at the base to spread out the wings and let the bird sit.